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INTRODUCTION

Game Theory is a powerful tool for Decision Making especially in the Oligopoly Markets where an interdependence of the sellers exists. With regard to that the purpose of this paper is twofold. First, it represents the main models of oligopoly as well as the nature of their strategic behavior; second, it discusses how the Theory of Games might be applied for strategic decisions making.

Strategic behavior refers to actions taken by firms which aim to influence the market environment in which they compete[1]. In regard to this definition, strategic behavior involves primarily long-run actions and decisions such as production capacity, research and development (R&D), investment, location, advertising, product differentiation.

On the other hand, in the economic theory[2] as well as the Game Theory, a **strategic action** is an action in which the company takes into account the expected reactions of its main rivals. In terms of this, strategic decisions might be defined in a shorter term as well. Throughout the paper we share this broader view of strategic behavior. Strategic actions could be divided into two main groups In accordance with the strength of competition, respectively interaction:

- actions which force rivals to act cooperatively so as to raise the joint profit such as various forms of collusion, including cartels
- non-cooperative actions which aim to raise the firm's profits at the expense of the rivals' profit; examples include price and non-price competition and predation, creation of artificial barriers to entry.

Strategic actions are more likely to occur in industries with a small number of buyers or sellers i.e. tight oligopolies[3]. The move of each company affects its rivals and their expected response must be kept in mind while shaping the best course of firm's actions. A common assumption of the non-cooperative oligopoly theory is that *each firm*

[1] See, OECD, Glossary of Industrial Organization Economics and Competition Law. See, www.oecd.org. This approach has been adopted in the field of Strategic Management as well as Economics of Industrial Organization.

[2] See, for ex. Salvatore (2004). As well, Geckil & Anderson (2010).

[3] A market structure is defined as a Tight Oligopoly if there are few large sellers – usually between 2 and 8 - whose aggregated market share is more than 60%.

chooses its strategy so as to maximize profits, given the profit-maximizing decisions of other firms[4].

When making a strategic decision the company takes into account the current and anticipated behavior of its major competitors. This is the **Game theory** which plays a central role here because:

- ✓ it allows to consider the interdependent nature of the oligopoly markets
- ✓ it is an useful tool for modeling the strategic behavior of the firm
- ✓ it clearly shows how the market outcome heavily depends on the strategy which each player adopts.

1. Fundamentals of Game Theory

Game theory is a relatively new branch of mathematics designed to help people who are in conflict situations determine the best course of action out of several possible choices. Even though the theory had its beginnings in the 1920's, its greatest advance occurred in 1944, when *John von Neumann* and *Oscar Morgenstern*, both at Princeton University, published their landmark book, *Games and Economic Behavior*.

1.1. Strategic form game

Games involving only two players and a payoff of some amount after each play such that one player's win is the other player's loss are called ***two-person zero-sum games.*** With the indicated restrictions, we will be able to determine the best strategies of play for each person.

The simplest mathematical description of a game is the ***strategic*** or ***normal form***. The strategic or normal form of a two-person zero-sum game is given by a triple (*X, Y, A*), where *X* is a nonempty set, the set of strategies of player 1; *Y* is a nonempty set, the set of strategies of player 2 and *A* is a real-value function defined on $X \times Y$. If both strategy sets *X* and *Y* are finite sets, a two-person zero-sum game is said to be a ***finite game***.

The interpretation is as follows. Simultaneously, player 1 chooses $x \in X$ and player 2 chooses $y \in Y$, each unaware of the choice of the other. Then their choices are made known and player 1 wins the amount *A(x, y)* form player 2. *A(x, y)* represents the winnings of player 1 and the losses of player 2.

[4] The microeconomic theory assumes that the company aims to maximize its economic profit. The economic profit is calculated as a difference between the accounting profit and the normal profit which is the alternative price of the company's own resources involved in production. More about the economic approach could be found, for example, in Pindyck, R., D. Rubinfield, Microecnomics. 6th Ed., Prentice-Hall of India, New Delhi, 2007. Regarding the accounting approach in Bulgaria, see Nikolova, G. Aspects of legislative regulation of the organization of accounting in the Republic of Bulgaria. Economics, Management and Financial Markets, vol. 9(1), Addleton Academic Publishers, 2014, pp. 394-402.

A finite two-person zero-sum game in a strategic form is called a ***matrix game*** because the playoff function *A* can be represented by a matrix.

Example 1.1. Let's consider two stories – *R* and *C* in a shopping mall that sell the same equipment. Each is trying to decide how to price its most competitive product. A market research firm supplies the following information:

$$
\begin{array}{ccc}
 & & \text{store } C \\
 & & \begin{array}{cc} 200euro & 225euro \end{array} \\
\text{store } R & \begin{array}{c} 200euro \\ 225euro \end{array} & \begin{bmatrix} 55\% & 70\% \\ 40\% & 55\% \end{bmatrix}
\end{array}
$$

The entries in the matrix indicate the market share which *R* will receive. For example, if store *R* chooses a price of 200 euro and store *C* chooses 225 euro, then store *R* will receive 70% of the business and store *C* will lose 70% of the business but will get 30%. Each store can choose its own price but cannot control the price of the other. The goal is to determine a price level that will ensure the maximum possible market share in this competitive situation. This situation may be viewed as a game between store *R* and store *C*. Store *R* will be a ***row player*** and store *C* will be a ***column player***. Each entry in matrix is called a ***payoff*** for a particular pair of moves by *R* and *C*. Matrix is called ***a matrix game*** or ***a payoff matrix.*** This game is a two-person zero-sum game, since each store may be considered a person, and *R* wins the same amount that *C* losses, and vice versa. Any $m \times n$ matrix may be considered a two-person zero-sum matrix game in which player *R* chooses any one of *m* rows while player *C* simultaneously chooses any one of *n* columns.

How should *R* and *C* play in a matrix game?

Example 1.2. Consider the following matrix

$$
\begin{pmatrix} 0 & 6 & -2 & -4 \\ 5 & 2 & 1 & 3 \\ -8 & -1 & 0 & 20 \end{pmatrix}
\begin{array}{c} -4 \\ 1^{*} \\ -8 \end{array}
$$

$$
\begin{array}{cccc} 5 & 6 & 1^{*} & 20 \end{array}
$$

Player *R* thinks in terms of the worst that could happen for each row choice. The worst that could happen in row 1 is a 4 unit loss, in row 2 – a 1 unit gain, and in row 3 – an 8 unit loss. Each of these values is in column 5. The best strategy for *R* is to select the row with the largest of these minimum values – row 2. With this choice, a win of at least 1 unit is guaranteed for *R* irrespective of *C*'s choices – ***security level.*** Similarly, *C* determines the worst situation that can happen in each column - the maximum value in each column. The best strategy is to select the column with the smallest of these maximum values – column 3. By choosing column 3 *C* establishes a security level of a 1 unit loss irrespective of *R*'s choices, and this is the best that *C* can do if *R* continues to make the best moves.

There are ***optimum strategies*** for both *R* and *C*. If *C* keeps playing the third column and *R* decides to change from the second row, then *R*'s wins cannot increase. Similarly, if *R* continues to play the second row and *C* deviates from the third column, *C*'s losses cannot decrease. The game is said to be ***determined*** in that *R* must always play row 2 to maximize its gains and *C* must always play column 3 to minimize losses. The payoff value is called a ***saddle point.*** Many matrix games do not have saddle values.

Example 1.3. *The classic penny-matching game* There are two players – *R* and *C*. Each player has a penny, and they simultaneously choose to show the side of the coin of their choice: *H*=heads and *T*=tails. If the pennies match, then *R* wins 1 euro. If pennies do not match, then *R* loses 1 euro. The following game matrix could be constructed:

$$\begin{array}{cc} & \textit{player} \quad C \\ & \begin{array}{cc} H & T \end{array} \\ \textit{player} \quad R \quad \begin{array}{c} H \\ T \end{array} & \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \end{array}$$

A minimum -1 occurs in each row, thus *R* could play. A maximum 1 occurs in each column, thus *C* could play either. There are no saddle values. The game is called ***non-strictly determined.*** In this type of game knowledge of the other player's move would be very useful.

Are there optimum strategies for each player in this case? The answer, surprisingly, is Yes. *R* and *C* should play each rows and column in some mixed pattern unknown to the other player. How should the mixed pattern be selected? The best way to choose a mixed pattern is to use a probably distribution. For example, *R* might choose row 1 with probability ¼ and row 2 with probability ¾. This means that in the long run *R* would play row1 one-fourth of the time and row 2 three-fourths of the time. Similarly, *C* might choose column 1 with probability 3/5 and column 2 with probability 2/5 by random.

Given the game matrix

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

R's strategy is denoted by a probability row matrix $P = (p_1, p_2), p_1, p_2 \geq 0, p_1 + p_2 = 1$

C's strategy is denoted by a probability column matrix $Q = (q_1, q_2), q_1, q_2 \geq 0, q_1 + q_2 = 1$

The strategy is called a ***pure strategy***, if one of the elements in *P* or *Q* is 1 and the other is 0. If a strategy is not a pure strategy, it is called a ***mixed strategy.***

The minimax theorem. For every finite two-person zero-sum game

- there is a number *v*, called the value of the game;

- there is a mixed strategy for player 1 such that player 1’s average gain is at least *v* no matter what player 2 does;
- there is a mixed strategy for player 2 such that player 2’s average loss is at most *v* no matter what player 1 does.

We find the optimal mixed strategy for player 1 by solving the following system:

$$\begin{vmatrix} a_{11}p_1 + a_{21}p_2 = v \\ a_{12}p_1 + a_{22}p_2 = v \\ p_1 + p_2 = 1 \end{vmatrix}$$

Similarly for player 2

$$\begin{vmatrix} a_{11}q_1 + a_{12}q_2 = v \\ a_{21}q_1 + a_{22}q_2 = v \\ q_1 + q_2 = 1 \end{vmatrix}$$

Example 1.4. Matrix game with payoff matrix

$$A = \begin{pmatrix} -4 & 3 \\ 6 & 0 \end{pmatrix}$$

doesn't have a saddle point. Solving the system

$$\begin{vmatrix} -4.p_1 + 6.p_2 = v \\ 3.p_1 + 0.p_2 = v \\ p_1 + p_2 = 1 \end{vmatrix}$$

we find the optimal mixed strategy for player 1 *P*=(6/13, 7/13), $v = 18/13$. Similarly,

$$\begin{vmatrix} -4.q_1 + 3.q_2 = v \\ 6.q_1 + 0.q_2 = v \\ q_1 + q_2 = 1 \end{vmatrix} \Rightarrow Q = (3/13,\ \ 10/13),\ v = 18/13$$

The simplest case to generalize two-person zero-sum games are ***the two-person non-zero-sum games***. The strategic form of this game is given by two sets *X* and *Y* of pure strategies of the players, and two real-valued function $u_1(x,y)$ and $u_2(x,y)$ defined on a $X \times Y$ matrix, representing the payoffs to the two players. A finite two-person game in strategic form can be represented as a matrix of ordered pairs, called a ***bimatrix***. The first component of the pair shows player1’s payoff and the second component represents player 2’s payoff.

Example 1.5. *Prisoners' dilemma*. Consider two prisoners Stan and Leland who have each been offered a deal to turn state's witness (defect) against the other. They can't communicate. The two players in the game can choose between two moves, either "cooperate" or "defect". The idea is that each player gains when both cooperate, but if only one of them cooperates, the other one, who defects, will gain more. If both defect, both lose (or gain very little) but not as much as the "cheated" cooperator whose cooperation is not returned. The whole game situation and its different outcomes can be summarized by the following payoff matrix:

Player B

Player A

	Solidarity (Cooperate)	Defection (Not cooperate)
Solidarity (Cooperate)	3, 3	1, 4
Defection (Not cooperate)	4, 1	1, 1

A strategy is designated as a ***dominant strategy*** if it holds for every other strategy that the latter do not put the player in a better position, and put him in a worse position in at least one case. A dominant strategy is thus resistant to any possible change of strategy by the opponent, and is selected in each instance.

In the Prisoners' dilemma, the strategy of defection is ***weakly dominant*** for each player, meaning that whatever the other player does, defecting yields an outcome at least as good and possibly better than remaining in solidarity would. Note that if the bottom right cell payoffs were (2,2) instead of (1,1), then defecting would be ***strictly dominant*** for each player. *Defection-Defection* has the property that each player's strategy is the best - Nash equilibria.

1.2. Nash equilibria

Nash equilibria are sets of strategies for players in a game such that no single one of them would be better off switching strategies unless others did.

Example 1.6. The company *I* is going to introduce a new camera into its product line and hopes to capture as large an increase in its market share as possible. In contrast, the company *II* hopes to minimize company *I*'s market share increase. The payoff table, which includes the strategies and outcomes for each company is shown in the next matrix

$$
\begin{array}{ccc} & & \textit{company} \quad I \\ & & \begin{array}{ccc} A & B & C \end{array} \\ \textit{company} \quad II & \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 9 & 7 & 2 \\ 11 & 8 & 4 \\ 4 & 1 & 7 \end{pmatrix} \end{array}
$$

The values in the above matrix are the percentage increases or decreases in market share for company *I*.

Step by step guide on how to find a Nash Equilibrium

Step1. Look at the payoff matrix and figure out the payoffs of each player

player B

player A		Cooperate	Not cooperate
	Cooperate	1500, 2000	50, 4000
	Not cooperate	2000, 100	60, 101

Step 2. Figure out Player A's best response to all of player B's actions

player B

player A		Cooperate	Not cooperate
	Cooperate	1500, 2000	50, 4000
	Not cooperate	2000, 100	60, 101

Since 2000>1500 and since 60>50.

Step 3. Figure out Player B's best response to all of player A's actions

player B

player A		Cooperate	Not cooperate
	Cooperate	1500, 2000	50, 4000
	Not cooperate	2000, 100	60, 101

Since 4000>2000 and since 101>100.

Step 4. A Nash equilibrium exists where Player B's best response is the same as Player A's best response

		player B	
		Cooperate	Not cooperate
player A	Cooperate	1500, 2000	50, 4000
	Not cooperate	2000, 100	60, 101

Using dominant strategies allows us to predict the outcome of a game. But in some games no player has a dominant strategy. We need a way to predict the outcome in this case. Such a way is given by the concept of Nash equilibrium. Nash equilibrium is a set of strategies, one for each player, such that each player is maximizing payoff given the other players' strategies. Thus, given what the other players are playing, a player has no incentive to deviate from its Nash equilibrium strategy. If every player has a dominant strategy, the set of dominant strategies is a Nash equilibrium, and it is the only Nash equilibrium of the game. In games where not every player has a dominant strategy, there may be more than one Nash equilibrium.

What predictions can we make about the next game? What about the existence of Nash equilibria in this case?

		player B	
player A		Cooperate	Not cooperate
	Cooperate	1, 3	0, 0
	Not cooperate	0, 0	3, 1

There are two Nash equilibria: (Cooperate, Cooperate) with payoffs (1,3), and (Not cooperate, Not cooperate) with payoffs (3,1). Here is why (Cooperate, Cooperate) is a Nash equilibrium. (The argument for (Not cooperate, Not cooperate) is entirely symmetric.) If player A expects player B to choose Cooperate, that's what he will choose as well, because the payoff of 1 is greater than a payoff of 0. And if player B expects player A to choose Not cooperate, he will choose the same because a payoff of 3 is greater than a payoff of 0.

Each Nash equilibrium is a theory of how the game should be played consistent with assumed rationality of the players and the mutual knowledge of that rationality. It seems plausible to predict that player A and his opponent will end up at a Nash equilibrium in this game, or at least that they ought to end up at a Nash equilibrium. That is, these rational players, in planning this game, would agree that the non-Nash outcomes are undesirable, and that the Nash equilibria, even though one is inferior to the other in each player's eyes, are reasonable in the sense that neither player would want to break an agreement to be at such an outcome.

player B

player A	Cooperate	Not cooperate
Cooperate	1, 3	4, -4
Not cooperate	2, -2	3, 1

When we examine the above payoff table, we observe one interesting pattern. From the upper left cell, player A would want to move down to the lower left cell. From the lower left cell, player B would want to move right to the lower right cell. From the lower right cell, player A would want to move up to the upper right cell. From the upper right cell, player B would want to move left to the upper left cell. In short, at every pair of strategies, one of the players would be unhappy and would want to change her or his strategy. Therefore, at least based on our definition of Nash equilibrium to this point, there is no Nash equilibrium in this game. In fact, our definition of Nash equilibrium up to now has assumed that a player can only choose a single strategy with certainty. Player A, for instance, can choose either "Cooperate" or "Not cooperate." If he chooses "Cooperate", he strictly follows this strategy. Choosing to cooperate for sure is called a *pure strategy*. The games we have been discussing to this point allow only pure strategies. But there is another way to play games like this. Players might make random choices over pure strategies.

Every game has a Nash equilibrium. If there is no Nash equilibrium in pure strategies, there is at least one Nash equilibrium in mixed strategies. A game can also have Nash equilibriums in pure and mixed strategies.

How is a mixed Nash equilibrium determined?

The equilibrium theorem. Consider a game with $m \times n$ matrix *A* and value *v*. let $\mathbf{p}=(p_1, p_2,\ldots,p_m)^t$ be any optimal strategy for player 1 and $\mathbf{q}=(q_1, q_2,\ldots,q_n)^t$ be any optimal strategy for player 2. Then

$$\sum_{j=1}^{n} a_{ij} q_j = v \text{ for all } i \text{ for which } p_i>0$$

and

$$\sum_{i=1}^{m} p_i a_{ij} = v \text{ for all } j \text{ for which } q_j>0.$$

Consider the following simple game: player 1 with two strategies – X_1 and Y_1 with probability *p* and 1-*p* correspondingly, and player 2 with two strategies – X_2 and Y_2 with probability *q* and 1-*q* correspondingly.

		player 2	
		X_2 (*q*)	Y_2 (1-*q*)
player 1	X_1 (*p*)	1, 2	2, 0
	Y_1 (1-*p*)	2, 0	0, 2

This game has no Nash equilibrium in pure strategies, but there exists at least one mixed Nash equilibrium. To determine it we need the probabilities with which a player has to play his strategies so that his opponent is indifferent with regard to these strategies. This means that I have to select my strategy in such a way that my opponent does not prefer a specific strategy with which he can outsmart me – which would then give me an incentive to change the strategy. Let player 1 considers which strategies he has to play so that player 2 is indifferent with regard to his strategies, i.e. the expected payoffs for player 2 must be equal for X_2 and Y_2.

$$E(A_2((p,1-p), X_2))= E(A_2((p,1-p), Y_2))$$

This equation can be determined in detail with the concrete probabilities and payoffs:

$$p.2+(1-p).0= p.0+(1-p).2 \Rightarrow p=1/2.$$

Let player 2 consider the same

$$E(A_1(X_1,(q,1-q)))= E(A_1(Y_1,(q,1-q)))$$

$$q.1+(1-q).2=q.2 \Rightarrow q=2/3$$

The probabilities of the mixed Nash equilibrium were now determined – S^*=((1/2, 1/2), (2/3, 1/3)).

Application to Business

		Firm B	
		Don't Invest	Invest
Firm A	Don't Invest	0, 0	-10, 10
	Invest	-100, 0	20, 10

Two firms compete over office application software. Because they use the same software standard, files used by one firm's software package could be read by the program developed by its rival – an advantage for consumers. Nonetheless, firm A has a larger market share because we suppose that it entered the market first. Now, both firms are considering investment in a new office package. The table represents the expected outcomes – profit in mln. USD – for the two alternative options each competitor has: to invest or not to invest.

Firm A should expect firm B to invest because this is its dominant strategy. Yet, if the manager of firm A tends to be cautious, and he is concerned that the manager of the rival firm might not be fully informed or rational, he might choose to play "don't invest". In that case the worst that could happen would be that he loses 10 mln.; he no longer has a chance of losing 100 mln. USD. Thus, the company follows a maximin strategy when maximizing the minimum gain that can be earned. If both firms follow maximin strategies the outcome would be that firm A does not invest, while firm B does. If firm A knew for certain that firm B follows a maximin strategy, it would prefer to invest instead of following its strategy of not investing.

If firm A could assign probabilities to each action of firm B, it would follow a strategy of maximum expected payoff. In this case, the decision of firm A crucially depends on the values of the probabilities. Suppose, for example, that firm A thinks that there is only a 10-percent chance that firm B will not invest. Then, its expected profit from investing is 0.1*(-100) + 0.9*20 = 8 million. On the other hand, the expected profit from not investing is negative: -9 mln. Therefore, firm A should invest. On the contrary, it could be found that if according to firm A there is a 30 percent chance that firm B will not invest, its decision will be completely different and it will not invest.

1.3. Extensive form games

Strategic form games are used to model situations in which players choose strategies without knowing the strategy choices of the other players. Most games we encounter in the real life are not in a strategic form. Players don't pick their entire strategies independently. Dynamic games are typically defined by their extensive form.

Extensive form games provide more information about the order of moves, actions available at different points in the game and information available throughout the game. The easiest way to represent an extensive form game is to use a ***game tree***. It's simply a diagram that shows that choices are made at different points in time (corresponding to each node). The payoffs are represented at the end of each branch.

- ***Decision nodes*** indicate a player's turn to move. The number inside a decision node indicates whose turn it is to move.
- ***Branches*** emanate from decision nodes. Each branch corresponds to an action available to a player at that node. The label for a branch corresponds to its action. Different nodes for the same player might have different sets of actions.
- The ***terminal nodes*** are the solid circles. These indicate that the game is finished. Beside each terminal node are the payoff for the two players if that node is reached. The first number is the payoff of the player who moved first.
- An ***information set*** of a player gives a set of that player's decision nodes which are indistinguishable to the player.

Example 1.7. Assume we have a two-player simultaneous game with two moves each. The normal form looks like follows:

	B_1	B_2
A_1	A_{11}, B_{11}	A_{12}, B_{12}
A_2	A_{21}, B_{21}	A_{22}, B_{22}

Then, as it is explained above, we remodel the game as a sequential game, where Ann decides first, and Beth, without knowing Ann's decision, has to decide next. That means that at Beth's decision, Beth has information set with two possible positions, where she does not know which one she is in when deciding. The extensive form then looks as shown below:

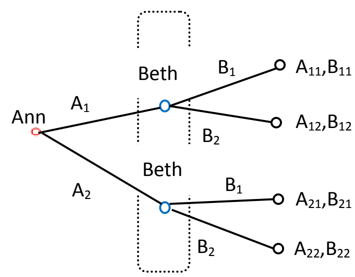

Figure 1.1. Extensive form - example 1.7

The strategy profile constructed by backward induction is a Nash equilibrium.

Backward induction

- Determine the optimal action(s) in the final stage K for each history h^K.
- For each stage j=K-1,…,1
  Determine the optimal action(s) in stage j for each possible h^j given the optimal actions determined for stages j+1,…,K.

Example 1.8.

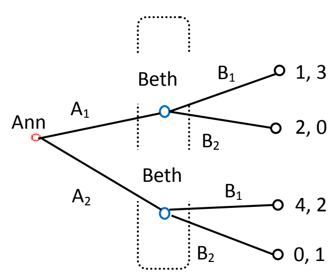

Figure 1.2. Unique equilibrium path

An extensive form game with ***perfect information*** is one for which all the information sets are singletons. This implies that each player is able to observe all previous moves or the entire history thus far. Each player knows precisely where he is currently and also knows precisely how he has reached that node. A game of ***imperfect information*** is one where some players do not know the entire history of the game when they go to move because they do not know some or all of the actions taken by their opponents earlier in the game.

Example 1.9. Consider a game with complete and perfect information. Let us conduct an analysis to determine the point of equilibrium in the Stakelberg's model (Anne – leader, Beth – follower). If the first player chooses his first strategy, the second should respond by choosing the second strategy. The profit is 1 and 2 respectively of the leader and follower. If the first player chooses the second strategy, the second should respond by choosing the first strategy. The profit is 2 and 1 respectively the leader and follower. After selecting your strategy leader knows the capabilities of the follower and decide what will be his answer. To maximize profit, he must choose the second strategy. Therefore, the equilibrium point in this model is (2, 1).

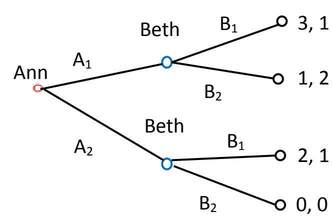

Figure 1.3. The point of equilibrium in the Stakelberg's model

1.4. Repeated games

In game theory, ***repeated games*** are those that are played out over and over for a period of time, and therefore are usually represented using the extensive form. As opposed to one-shot games, repeated games introduce a new series of incentives: the possibility of cooperating means that we may decide to compromise in order to carry on receiving a payoff over time, knowing that if we do not uphold our end of the deal, our opponent may decide not to either.

Note that in a repeated game we must distinguish between *actions* and *strategies.*

- Actions are the choices available to a player when it is his turn to move.
- A strategy specifies the action a player will take at each of his decision nodes. A strategy specifies how a player will behave at a node even if during actual play the node is not reached. A strategy in a game is a complete-contingent plan.

When we say that in each period we're playing the same stage game this means that:

a/ For each player the set of actions available to her in any period in the game G is the same regardless of which period it is and regardless of what actions have taken place in the past.

b/ The payoffs to the players from the stage game in any period depend only on the action profile for G which was played in that period, and this stage-game payoff to a player for a given action profile for G is independent of which period it is played.

These statements are saying that the environment for our repeated game is *stationary*.

In order to define a repeated game, first we must specify the players' strategy spaces and payoff functions. Every *n*-player normal form game is a game that is repeated and is called "a stage game". The stage game is played at each discrete time period $t=0, 1, 2,\dots, T$ and at the end of each period, all players observe the realized actions. The game is ***finitely repeated*** if $T<\infty$ and ***infinitely repeated*** otherwise.

Example 1.10. *Repeated Prisoner's dilemma*. In the game known as Prisoner's dilemma, the Nash equilibrium is defect-defect. In order to see what equilibrium will be reached in a repeated game of the prisoner's dilemma, we must analyze two cases: 1). the game is repeated a finite number of times and 2). the game is repeated an infinite number of times.

When the prisoners know the number of repetitions, it's interesting to operate a backwards induction to solve the game. The set of equilibria of an infinitely repeated game can be very different from that of the corresponding finitely repeated game because players can use self-enforcing rewards and punishments that do not unravel from the terminal date. Consider the strategies of each player when they realize the next round is going to be the last. They behave as if it was a one-shot game, thus the Nash equilibrium applies, and the equilibrium would be defect-defect, just like in the one-time game. Now consider the game before the last. Since each player knows in the next, final round they are going to defect, there's no advantage to lie (cooperate with each other) on this round either. The same logic applies for prior moves. Therefore, defect-defect is the Nash equilibrium for all rounds.

The situation with an infinite number of repetitions is different, since there will be no last round, backwards induction reasoning does not work here. At each round, both prisoners guess there will be another round and therefore there are always benefits arising from the cooperative (lie) strategy. However, prisoners must take into account punishment strategies, in case the other player confesses in any round. ***Punishment strategy*** is a strategy used in a repeated game to secure an outcome which is not a Nash equilibrium for a single play of the game.

While in finitely repeated games the strategy can explicitly state what to do in each of the T periods, specifying strategies for infinitely repeated games is more tricky because it must specify actions after all possible histories, and there is an infinite number of these. Here are the specifications of several common strategies:

1. Tit-For-Tat - Repeat opponent's last choice.

2. Tit-For-Tat and Random - Repeat opponent's last choice skewed by random setting.
3. Tit-For-Two Tats and Random - Like Tit-For-Tat except that opponent must make the same choice twice in a row before it is reciprocated. Choice is skewed by random setting.
4. Tit-For-Two Tats - Like Tit-For-Tat except that opponent must make the same choice twice in row before it is reciprocated.
5. Naive Prober (Tit-For-Tat with Random Defection) - Repeat opponent's last choice (i.e. Tit-For-Tat) but sometimes probe by defecting in lieu of co-operating.
6. Remorseful Prober (Tit-For-Tat with Random Defection) - Repeat opponent's last choice (i.e. Tit-For-Tat), but sometimes probe by defecting in lieu of co-operating. If the opponent defects in response to probing, he shows remorse by co-operating once.
7. Naive Peace Maker (Tit-For-Tat with Random Co-operation) - Repeat opponent's last choice (i.e. Tit-For-Tat), but sometimes make peace by co-operating in lieu of defecting.
8. True Peace Maker (hybrid of Tit-For-Tat and Tit-For-Two Tats with Random Co-operation) - Co-operate unless opponent defects twice in a row, then defect once, but sometimes make peace by co-operating in lieu of defecting.
9. Random - always set at 50% probability.
10. Always Defect
11. Always Co-operate
12. Naive Grim Trigger – cooperating while the other player cooperates and should the other player defect even once, defect forever there-after.
13. Grim Trigger – cooperating in the initial period and then cooperating as lon as both players cooperated in all previous periods.
14. Grudger (Co-operate, but only be a sucker once) - Co-operate until the opponent defects. Then always defect unforgivingly.
15. Pavlov (repeat last choice if good outcome) - If 5 or 3 points scored in the last round then repeat last choice.
16. Pavlov / Random (repeat last choice if good outcome and Random) - If 5 or 3 points scored in the last round then repeat last choice, but sometimes make random choices.
17. Adaptive - Starts with c,c,c,c,c,c,d,d,d,d,d and then takes choices which have given the best average score re-calculated after every move.
18. Gradual - Co-operates until the opponent defects, in such case defects the total number of times the opponent has defected during the game. It is followed up by two co-operations.
19. Suspicious Tit-For-Tat - As for Tit-For-Tat except begins by defecting.
20. Soft Grudger - Co-operates until the opponent defects, in such case opponent is punished with d,d,d,d,c,c.
21. Customised strategy 1 - default setting is T=1, P=1, R=1, S=0, B=1, always co-operate unless sucker (ie 0 points scored).
22. Customised strategy 2 - default setting is T=1, P=1, R=0, S=0, B=0, always play alternating defect/co-operate.

Everything is straightforward if we repeat a game a finite number of times we can write the whole thing as an extensive-form game with imperfect information (at each round players don't know what the others have done). The overall payoff function is a sum of payoffs in stage games.

Example 1.11.

		player 2	
		C (cooperate)	D (defection)
player 1	C (cooperate)	-1, -1	-4, 0
	D (defection)	0, -4	-3, -3

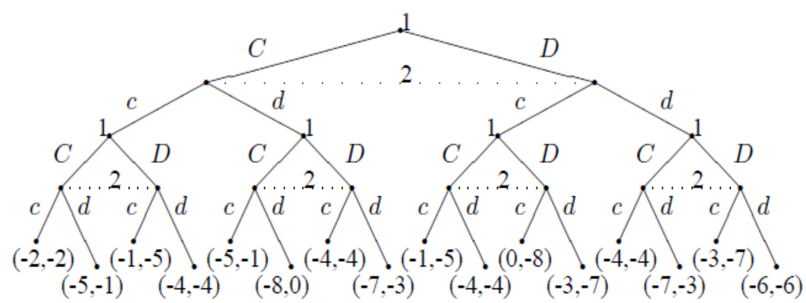

Figure 1.4. The overall payoffs

1.5. Game against nature

Decision making under uncertainty is a situation where the decision maker has to decide, but some information is missing or it is given with a probability of some events. The goal is to pick the best (optimal) solution. A frequently used model for such situations is ***a game against nature***. In this model, ***nature*** is called a formal description of the conditions under which the decision was taken.

Game Against Nature is a 1-player game, in which a single rational self-interested player must choose a strategy, and the outcome and the player's *payoff* depends on both his chosen strategy and the "choice" made by a totally disinterested *nature*. Whatever the empirical values contained in the payoff matrix, nature's strategy for any single play of the game may be considered as being predetermined and unalterable. In the absence of knowledge about which strategy nature will play, we may use the principle of insufficient reason and assume that all state of nature are equally likely. On the other hand, if an a priori or a posteriori probability distribution over the various states of nature is available, the decision problem under uncertainty is converted to one under risk.

The rule for selecting the optimal solution is called a ***criterion***. It compares each possible solution of choice to some number. Depending on the meaning of this number the solution is optimal for which this number is either the greatest or the smallest.

The Wald criterion. If the player were continually to play his i^{th} strategy, then his expected payoff would at least be equal to the smallest element in the i^{th} row. Using the minimax principle, the decision maker attempts to maximize this smallest possible payoff, i.e. he selects that row for which $\min_j a_{ij}$ is maximized.

The Savage criterion. Savage has suggested that the decision maker attempts to minimize the regret. The regret matrix is consisted of element r_{ij}, which is defined as the amount that has to be added to a_{ij} to equal the maximum payoff in the j^{th} column. The decision maker should then choose that row for which $\max_j r_{ij}$ is minimized.

The Hurwicz criterion. Using this criterion, the decision maker constructs an index for each of his strategies which takes into account both the best and worst possible outcomes from using each strategy, weighted by a constant which measures the degree of his pessimism or optimism. Let ε, a constant between 0 and 1, measures the player's level of pessimism. The Hurwicz criterion leads the player to select that row for which the "pessimism-optimism index" is

$$\max_i \left\{ \varepsilon \min_j a_{ij} + (1-\varepsilon) \max_j a_{ij} \right\}, \varepsilon \in (0,1) .$$

Example 1.12. Given a game against nature with a matrix of payoff tables 2.1 and 2.2 are provided as output information to it, and the results of the criteria considered.

Table 1.1. Wald criterion and Hurwicz criterion

A_i \ Π_j	Π_1	Π_2	Π_3	*Wald criterion* $\alpha_i = \min_j a_{ij}$	$\omega_i = \max_j a_{ij}$	*Hurwicz criterion* $h_i = \varepsilon \min_j a_{ij} + (1-\varepsilon) \max_j a_{ij}$ $(\varepsilon = 0,6)$
A_1	0,20	0,30	0,15	0,15	0,30	0,21
A_2	0,75	0,20	0,35	0,20	0,75	0,42
A_3	0,25	0,80	0,25	0,25*	0,80	0,47*
A_4	0,85	0,05	0,45	0,05	0,85	0,37

Table 1.2. Savage criterion

A_i \ Π_j	Π_1	Π_2	Π_3	*Savage criterion* $\gamma_i = \max_j r_{ij}$
A_1	0,65	0,50	0,30	0,65
A_2	0,10	0,60	0,10	0,60*
A_3	0,60	0	0,20	0,60*
A_4	0	0,75	0	0,75

2. Strategic Decisions of Oligopoly

Game Theory is a useful tool for strategic decision making in oligopoly markets. Therefore, it is crucial to understand the logics of the oligopoly models. This section focuses on the standard models considered by the economic theory such as the models of Cournot and Bertrand, the Stackelberg model of two-stage competition as well as the most popular form of a collusive oligopoly – cartel[5]. For each case we will try to derive the **optimal solutions** for the output and the selling price which maximize the firm's profit given the profit-maximizing decision of its rivals.

2.1. Cournot Model

The Cournot model of oligopoly is named after the French mathematician Augustin Cournot who tried to explain the optimal behaviour of a duopoly producing a homogeneous product. The model assumes that:

1. There are two identical firms in the industry – a symmetric duopoly
2. Companies produce a homogenous product, and each attempts to maximize profits by choosing how much to produce i.e. the companies compete in quantities.
3. Firms determine the production output simultaneously; each firm chooses its profit maximizing quantity, taking as given the quantity of its rival.

[5]The oligopoly models are discussed in the courses of Microeconomics, Managerial Economics as well as the Theory of Industrial Organization.

The oligopoly models might be solved analytically using economic reasoning and graphical presentations as well as numerically using formulas and mathematical transformations. The next paragraph illuminates the logics behind the *numerical solution* of the classical Cournot model in its simplest form – a linear demand function and constant marginal and average costs[6].

Let's suppose that the market demand has the following form:

$$\mathrm{D{:}\,P} = \mathrm{a} - \mathrm{b} * \mathrm{Q} = \mathrm{a} - \mathrm{b} * (q_1 + q_2) \qquad (2.1)$$

The constant *a* represents the maximum price which buyers are willing and able to pay for the product – the so called "reservation price" – while the positive parameter *b* shows by how much the price should decrease in order to sell one additional unit of the product[7].

The total quantity supplied by the market is denoted by Q, where Q is a sum of the quantity supplied by firm 1 (q_1) and the quantity supplied by firm 2 (q_2).The function of the total costs is the same for both companies and it is given by:

$$TC_i = \mathrm{c}q_i, \quad \mathrm{i} = 1,2 \qquad (2.2)$$

Such a function implies that marginal costs (MC), average variable costs (AVC) as well as average total costs (AC) are equal to *c* (MC = AVC = AC = c)[8].

The quantity which maximizes the profit of each firm *i* is found by solving the equation:

$$\mathrm{MR}i\,(q_i) = \mathrm{MC}i\,(q_i), \quad i = 1,2 \qquad (2.3)$$

Therefore, we must solve a system of two equations each one representing one of the firms[9]. Starting with firm 1 we could express its total revenue (TR_1) as follows:

$$TR_1 = P * q_1 \qquad (2.4)$$

Replacing *P* by the demand equation (1) we get:

$$TR_1 = P * q_1 = \big(a - b(q_1 + q_2)\big) * q_1 \qquad (2.5)$$

After some math transformations the equation takes the form:

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[6] Following the algorithm presented below the numerical solution could be found for any other form of market demand as well as total cost function.

[7] $b = \frac{\Delta \mathrm{P}}{\Delta Q}$. It shows the change in the product price when the quantity demanded increases by one unit.

[8] The marginal cost of the last unit produced could be found as a first derivative of the total costs function $TC(q_i)$. In this simplest case, the fixed costs are zero, therefore marginal costs (MC), average variable costs (AVC) as well as average total costs (AC) equal *c* (MC = AVC = AC = c).

[9] MR is the marginal revenue from the last unit produce. It could be found as the first derivative of the total revenue function $TR(q_i)$.

$$TR_1 = -bq_1^2 + aq_1 - bq_2q_1 \qquad (2.6)$$

The marginal revenue (MR_1) is found by taking the first partial derivative.

$$MR_1 = -2bq_1 + a - \ bq_2 \qquad (2.7)$$

By introducing (2.7) into (2.3) and replacing MC by c we get the resulting equation for the profit maximizing quantity:

$$-2bq_1 + a - bq_2 = c \qquad (2.8)$$

On the basis of this linear relation we could derive the firm's reaction curve**.** It shows how many units (q_1) the profit maximizing company should produce given an assumption for the quantity produced by its rival.

$$q_1 = \frac{a-c-bq_2}{2b} \qquad (2.9)$$

In case of equal marginal costs, firm 2's reaction curve looks in an identical way:

$$q_2 = \frac{a-c-bq_1}{2b} \qquad (2.10)$$

Solving the system (2.9) and (2.10) for the two unknowns we find the quantities supplied in equilibrium.

$$q_1^* = \ q_2^* = \frac{a-c}{3b} \qquad (2.11)$$

In equilibrium each company produces the quantity which its competitor suggests therefore both maximize their economic profit. In this example, the optimal quantities hence profits are the same because we suggested equal marginal costs. The total market supply is a sum of q_1 and q_2 – the production quantities of the sellers.

$$Q^* = q_1^* + q_2^* \ = \frac{2(a-c)}{3b} \qquad (2.12)$$

The market clearing price is calculated from the Law of Demand (2.1) replacing Q by (2.12). This makes it possible to estimate the total revenues, total costs and the firm profit as it is shown in table 2.1.

It is worth comparing the Cournot model with the case in which the companies collude and act as if there is one company which satisfies the whole market demand. By applying a similar algorithm we could calculate supply, price and profit. Following (2.1) we find that:

$$\mathrm{TR} \ = \ \mathrm{P} \ * \ \mathrm{Q} \ = \ (\mathrm{a} - \mathrm{b} * \mathrm{Q}) \ * \ \mathrm{Q} \ = \ \mathrm{a} * \mathrm{Q} - \mathrm{b} * Q^2 \qquad (2.13)$$

The profit maximizing equation is MR = MC which is equivalent to:

$$\mathrm{a} - \ 2 * \mathrm{b} * \mathrm{Q} \ = \ \mathrm{c} \qquad (2.14)$$

It is easily found that if companies act as a monopoly, the overall production would be less than that in case of Cournot competition. Moreover, it will be equally divided among the rivals.

$$Q_M^* = \frac{a-c}{2b}, q_{1M}^* = q_{2M}^* = \frac{a-c}{4b} \quad (2.15)$$

Table 2.1 represents the market outcome of the Cournot game vs. monopoly or cartelization[10]. It is assumed that the Law of Demand takes the form P = 120 – Q, while the total costs per seller are represented by: $TC_i = 30q_i$, $i = 1, 2$.

It is not unexpected that if the two companies succeed in making an agreement, consumers would buy less at the expense of a higher price which would end in higher profit levels.

Table 2.1. Cournot output vs. Monopoly/Collusive Output

	Cournot model	Cartel
Quantity supplied by firm 1: q_1^*	30	22.5
Quantity supplied by firm 2: q_2^*	30	22.5
Market supply: $Q = q_1^* + q_2^*$	60	45
Market Price: P = 120 – Q	60	75
$TR_1 = TR_2 = P*q$	1800	1687.5
$TC_1 = TC_2 = 30*q$	900	675
Firm profit: $Pr_1 = Pr_2 = TR - TC$	900	1012.5

The model of Cournot gives rise to results which are of great importance for industrial economics.

✓ As the number of firms increases, the equilibrium approaches that of perfect competition (see, Table 2.2). On the other hand, as concentration rises, industry performance deviates more from the norm of perfect competition.

[10] In case of cartelization a monopoly outcome appears only if the cartel includes all companies in the industry.

✓ Price will not in most cases equal marginal costs Moreover, the degree to which each firm's price exceeds marginal cost is directly proportional to the firm's market share and inversely proportional to the market elasticity of demand as well as the number of sellers.

Table 2.2 Optimal Solutions of Cournot according to the number of sellers[*]

Number of Companies in the Industry	2	3	4	N
Quantity Supplied by Each Firm[**]	$\frac{Q_c}{3}$	$\frac{Q_c}{4}$	$\frac{Q_c}{5}$	$\frac{Q_c}{N+1}$
Market Supply[***]	$\frac{2Q_c}{3}$	$\frac{3Q_c}{4}$	$\frac{4Q_c}{5}$	$\frac{N \cdot Q_c}{N+1}$

[*] It is supposed that the firm production costs are the same for all companies in the industry.
[**] Qc is the market supply for perfect competition.
[***] The price could be found on the basis of the Law of Demand and the quantity supplied.

The Cournot behavior is represented by a static one-shot game which is introduced in section 1.1. The Nash equilibrium defines the equilibrium outcome which is not the collusive outcome.

2.2. Stackelberg Model: The first-mover advantage

Stackelberg's model has been developed as an extension of the model of Cournot. In the same vein, it assumes that the industry consists of two firms which take decisions on the production output. The difference is that they do not act simultaneously therefore, this is an example of a sequential game. The logical reasoning of the model is as follows: one company – a leader – choses quantity first; the other firm – a follower – observes the leader's quantity and then defines the quantity supplied; the market price is set to clear the market. Contrary to the numerical solution presented above the Stackelberg model is solved using the backward induction i.e. we define the quantity supplied by the follower first, then the quantity produced by the leader.

In order to make the comparison of the models easier, we are going to assume the same market demand:

$$\mathrm{D{:}\ P} = \mathrm{a} - \mathrm{b} * \mathrm{Q} = \mathrm{a} - (q_L + q_F) \qquad (2.16)$$

The follower chooses the quantity which maximizes his profit given the choice of the leader. Consequently, its total revenue should be expressed as a function of the quantity supplied by the leader. Thus, the model is being solved by backward induction.

$$\mathrm{TR}_F = \mathrm{P} * q_F = (\mathrm{a} - \mathrm{b} * (q_L + q_F)) * q_F \qquad (2.17)$$

$$\mathrm{TR}_F = \mathrm{a} * q_F - \mathrm{b} * q_L * q_F - \mathrm{b} * q_F^2 \qquad (2.18)$$

Again, the optimal quantity is to be found by equating marginal revenues and marginal costs. Taking in mind that MC = c[11], we could write down the optimal condition for the follower $MR_F = MC_F$ i.e.

$$\mathrm{a} - \mathrm{b} * q_L - 2 * \mathrm{b} * q_F = \mathrm{c} \qquad (2.19)$$

The optimal quantity of the follower is:

$$q_F = \frac{a-c-bq_L}{2b} \qquad (2.20)$$

Expressions (2.20) and (2.9) look alike. Next, we are ready to find the solution for the leader taking into consideration that he knows how the follower's respond.

$$TR_L = \mathrm{P} * \mathrm{q}_L = (\mathrm{a} - \mathrm{b} * (\mathrm{q}_L + \mathrm{q}_F)) * q_L \qquad (2.21)$$

Introducing (2.20) into (2.21), we get:

$$TR_L = \frac{(a+c-bq_L)}{2} q_L \qquad (2.22)$$

The first order condition for the leader ($MR_L = MC_L$) results in:

$$q_L^* = \frac{a-c}{2b} \qquad (2.23)$$

The quantity maximizing the follower's profit is:

$$q_F^* = \frac{a-c}{4b} \qquad (2.24)$$

If P = 120 – Q, while MC = 30, the leader will supply q_L^* = 45 units, while the follower will produce half that quantity i.e. q_F^* = (90 – q_L)/2 = 22.5 units. The overall supply is expected to be 67.5 units solved at $52.5 each.

A crucial implication of the model is that *the leader takes the advantage of moving first* because it produces larger quantity and gains more than the follower. Often it is called a "first-mover" advantage. It arises when the industry leader takes its strategic decisions first thus gaining a greater profit. In regard to efficiency, the market output is larger than that observed in the Cournot equilibrium which in case of constant marginal costs improves efficiency. However, if the larger firm is inefficient, the larger output might not

[11] We assume the same total costs function as that shown in section 2.1 i.e. $TC_i = cq_i$.

lead to a more efficient outcome. The Stackelberg game is modeled by extensive form games represented in section 1.3.

2.3. Bertrand Model: Price competition

A main drawback of the Cournot model is the assumption that companies compete in supply. The Bertrand model assumes that firms independently choose prices – instead of quantity supplied – in order to maximize profits.

The model is suitable for describing markets with the following features:

- ✓ A duopoly producing a homogeneous product in which each company aims at maximizing the short-run profit
- ✓ Each firm decides on what price to charge; it determines the pricing policy simultaneously and independently of its rival
- ✓ The marginal costs are constant and different from zero.

The firm first defines the selling price which maximizes its profit then decides how much it would be able to sell at that price. In case of identical (homogeneous) products buyers would prefer the cheaper product. Thus, the price will be the main determinant of the buyers' choice. If a firm sets a price higher than that of its rival, the quantity sold would equal zero. On the other hand, if the price is lower than the selling price of its competitor, it will get the whole market demand. The Bertrand outcome is derived usually by economic reasoning.

1). If the selling price is less than the marginal costs, it would incur loses. Thus, no company has an incentive to set $P < MC$ because it will shut down.

2). If one firm, say firm 1, sets a price level higher than its marginal costs ($P_1 > MC$), it expects positive profit. Taking into consideration the price sensitivity of the buyers, its competitor – firm 2 – would price its product down in order to get the whole market. It could sell at price P_2 which is higher than MC and less than P_1. This means than no firm has an incentive to sell at price higher than its rival as well as the marginal costs.

The logical conclusion is that the equilibrium of the classical Bertrand model is such that the price equals marginal cost ($P_1 = P_2 = MC$), whereas the market supply is equally split between the two rivals. This means that the duopoly market looks just like perfect competition. The result holds regardless of the number of firms and stands in contrast to the Cournot equilibrium where the deviation from the competitive outcome increases as the number of sellers decreases.

The Bertrand game is also characterized by the Prisoners' Dilemma because if the firms try to coordinate their pricing policies, but set the prices independently, each of them has an incentive to cheat. By lowering its price the company increases its profit. At equilibrium both are worse off than if they were able to sell at a monopoly price.

It should be noted that this perfectly competitive result – the so called "Bertrand paradox" – appears only if the abovementioned assumptions hold. The prices are expected to exceed marginal costs in case of:

- ✓ product differentiation – If firms sell differentiated products the price competition does not necessarily drives the price down to the level of marginal costs; in this case the lower price does not guarantee that the company will "steal" all customers from its rivals
- ✓ capacity constraint – If the aggregate production capacity of the companies is lower in relation to the market demand, the lower supply would drive the price upward.

The static models allow for better understanding of the long-run strategic behavior such as product development, capacity, advertising, etc. An important result of the static theory of oligopoly is that the equilibrium outcome is not a collusive outcome that is the prices and profits are lower than those in case of monopoly. Therefore, these models provide the foundation for dynamic models and the repeated long-run collusion among sellers which aims at achieving a monopoly market position and power.

Business applications of the models

The Cournot model is suitable for markets in which the companies set its production capacity first, then, define the market clearing price according to the Law of Demand. On the other hand, if the companies initially set the price then adjust their quantities the Bertrand model is more appropriate. Examples for the latter are software products, services, publishing, while examples of the former are: tourist sector - hotels, restaurants, industry.

The Stackelberg model best represents the dominant firm structure[12] where the biggest firm in the industry first defines its production capacity. In case of very low marginal costs it might be profit maximizing for it to produce at that capacity. Then, the smaller rivals take the quantity supplied as given and produce their optimal output. This is an example of strategic behavior as long as the leader turns business environment to its advantage.

2.4. Cartels

In both models described above the oligopolies do not receive the maximum attainable profit due to the competition in either the total output or the price. By coordinating their actions the firms aim at achieving a monopoly market position. The centralized cartel is a market structure at which producers of a given product explicitly agree on: the selling price of the product, the total quantity supplied as well as the quota

[12] According to the OECD definition, the dominant firm model assumes the existence of one large company in a given industry with at least 40% market share. See, www.oecd.org.

of each member[13]. The objective of the cartel is rather long-run profit maximization than short-run benefits.

The cartel aims at achieving the highest level of both the overall profit and the profit of each participant. The equilibrium quantities are defined according to the following rules:

1). The total quantity supplied is defined according to the rule:

$$MR_Q = \mathrm{MC}_Q,$$

where MR_Q is the marginal revenue for the whole cartel, while MC_Q denotes the marginal cost for the cartel. MR_Q and MC_Q are derived from the total market demand and the aggregate production cost calculated as a sum of the costs of all members. If it includes all market players, the cartel behaves like a pure monopoly. Section 2.1 specifically (2.13) – (2.15) show how the monopoly optimal output is being calculated.

2). Each company produces the quantity for which the marginal revenue (MR_Q) equals the firm's marginal cost (MC_{q_i}). The quotas satisfy the condition:

$$MR_Q = MC_{q_1} = MC_{q_2} = \cdots = MC_{q_n}$$

3). The selling price equates the cartel total demand and the total quantity produced by the cartel.

Table 2.3. Comparison of the models with regard to the market supply

Type of the model	*Market Supply*
Perfect Competition	Qc
Bertrand Duopoly (Price Competition)	Qc
Stackelberg Duopoly (a Leader and a Follower)	$\frac{3}{4}Qc$
Cournot Duopoly	$\frac{2}{3}Qc$
Pure Monopoly or Cartel including all firms	$\frac{1}{2}Qc$

[13] This is only one of the forms of cartelization. The most popular types include a market sharing cartel, non-price agreements, collusive tendering/bidding, etc. Classic textbook examples of successful cartels are the Organization of Petroleum Exporting Countries (OPEC) and DeBeers.

Although the collusion often leads to monopoly profit, the companies sometimes escape it. The main reasons which hinder the long-run cooperation comprise:

- ✓ Sustainability of the agreement and the stimulus for cheating: That crucially depends on the punishment strategy as well as the strength and credibility of penalty for cheaters. Section 1.4 specifies a number of such strategies as grim trigger strategy and tit-for-tat strategy are being widely used. Another important point is whether the prices are observable. If not, the companies have a greater incentive for price cuts
- ✓ Rate and speed of entry and exit in the industry – In markets with a high rate of entry and exit of sellers the probability that a firm leaves the industry in the next period is large. Such a firm would prefer taking a higher profit by cheating today to following the agreement for there is little or nothing to lose.
- ✓ Duration of the agreement – The longer the period of the agreement the less the stimulus to deviate from it.
- ✓ Antitrust policy and extend of fines for collusion.
- ✓ Cartelization is represented by repeated games introduced in section 1.4.

Figure 2.1. Comparison of the Market Quantity and Prices

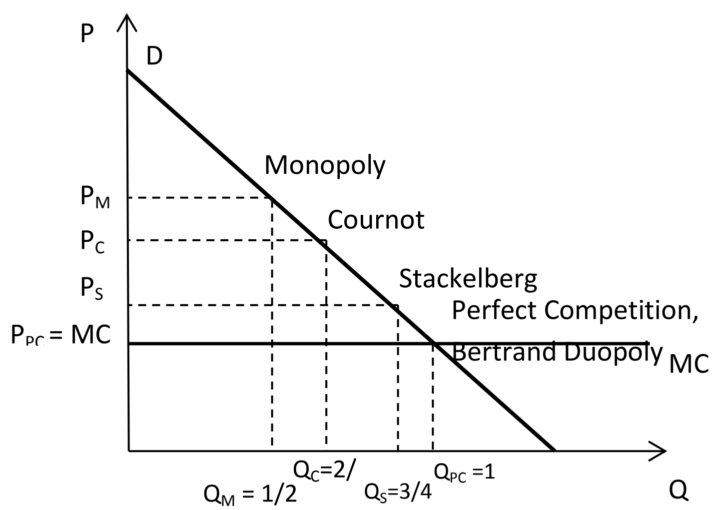

Table 2.3. and figure 2.1. shed light on the market outcome for the abovementioned models. As it is clearly seen, perfect competition and Bertrand duopoly lead to the highest output denoted by *Qc*. In case of a market leader and a follower - the model of Stackelberg – the firms supply 75% of the perfectly competitive markets at a higher price. If the companies compete in production the Cournot competition prevails and the buyers face lower supply – 2/3[rd] of *Qc*. Not surprisingly, the pure monopoly implies the lowest quantity – 50% of the competitive market output – at the highest price.

CONCLUSION

The Theory of Games is a useful tool for outlining the strategic actions of oligopoly because a key feature of this market is the interdependence of the leading players. In order to apply it properly one must take into account:

- The model which best describes the type of competition between sellers, for example: Cournot competition if companies define the production capacities and compete in quantities, Bertrand model in case of price competition or cartel agreement, etc.
- The type of game which corresponds to the oligopoly model i.e. one-shot static game, dynamic repeated game, extensive form game, etc.
- The expected company's profit for a specific strategy of its rival.

The knowledge of Game theory helps managers: 1). To find the best long-run strategy which will change the market environment in their behalf; 2) To find the optimal course of short-run tactical actions in terms of the current business conditions. Its application is facilitated by software programs developed to find specific solutions.

References

Cabral, L., Introduction to Industrial Organization. PHI Learning Private Limited, 2010.

Church, J., R. Ware, Industrial Organization: A Strategic Approach. McGraw-Hill, USA, 2000

Geckil, I, P. Anderson, Applied Game Theory and Strategic Behavior., Taylor and Francis Group, US, 2010.

Pindyck, R., D. Rubinfield, Microecnomics. 6th Ed., Prentice-Hall of India, New Delhi, 2007.

Salvatore, D. Managerial Economics in a Global Economy. 5th Ed., Thomson South-Western, 2004.

Shapiro, C., Theories of Oligopoly Behavior in R. Schmalensee and R. Willig (eds), The Handbook of Industrial Organization, North Holland, Amsterdam, 1989.

Varian, H, R., Intermediate Microeconomics: A modern Approach. 8th Ed., W.W. Norton & Company, USA, 2000.

Webster, T., Managerial Economics: Theory and Practice. Academic Press/Elsevier, USA, 2003.

APPENDIX

Game Theory for Business Decision Making: An Integrated Case Study

The following case study[14] combines the Theory of oligopolies, specifically the models of Cournot and Stackelberg, with the Theory of Game.

Two firms produce natural leather shoes: NL Shoes (NLS) and Best Leather Quality (BLQ). Each firm has a cost function given by:

$$\mathrm{TC}\ (\mathrm{q}) \ = \ 30\mathrm{q}, \text{ q is the firm's quantity supplied}$$

The market demand is represented by:

$$\mathrm{P} \ = \ 300 - \ 3\mathrm{Q}, \text{ Q is the market supplied, } \mathrm{Q} = \mathrm{q}_1 + q_2$$

(i) If each firm acts to maximize its profit, taking its rival's output as given (i.e. the firms behave as Cournot oligopolists), what will be the equilibrium quantities selected by each firm? What is the total output, and what is the market price? What are the profits for each firm?

(ii) It occurs to managers of NLS and BLQ that they could do a lot better by colluding. If the two firms collude, what will be the profit-maximizing choice of output? The industry price? The output and the profit for each firm?

(iii). The managers of these firms realize that explicit agreements to collude are illegal. Each firm must decide on its own whether to produce the Cournot quantity or the cartel quantity. To aid in making the decision, the manager of NLS constructs a payoff matrix like the one below. Fill in each box with the profit of NLS and the profit of BLQ. Given this payoff matrix, what output strategy is each firm likely to pursue?

Profit Payoff Matrix (NLS Profit, BLQ Profit)		BLQ	
		Produce Cournot q	Produce Cartel q
NLS	Produce Cournot q		
	Produce Cartel q		

(iv). Suppose NLS can set its output level *before* BLQ does. How much will each firm decide to produce? Find the market price and the profit for each player. Are there any benefits of choosing the output first? Explain. Compare the market output in the three cases considered – a Cournot game, a Stackelberg game and a cartel agreement.

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[14] The case study is adapted from Pindyck & Rubinfeld (2007).