

الأكاديمية العربية الدولية



الأكاديمية العربية الدولية المقررات الجامعية



مشروع الطرق المؤدية إلى التعليم العالي

مركز تطوير الدراسات العليا والبحوث
كلية الهندسة – جامعة القاهرة



Cairo University

إعداد
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القاهرة
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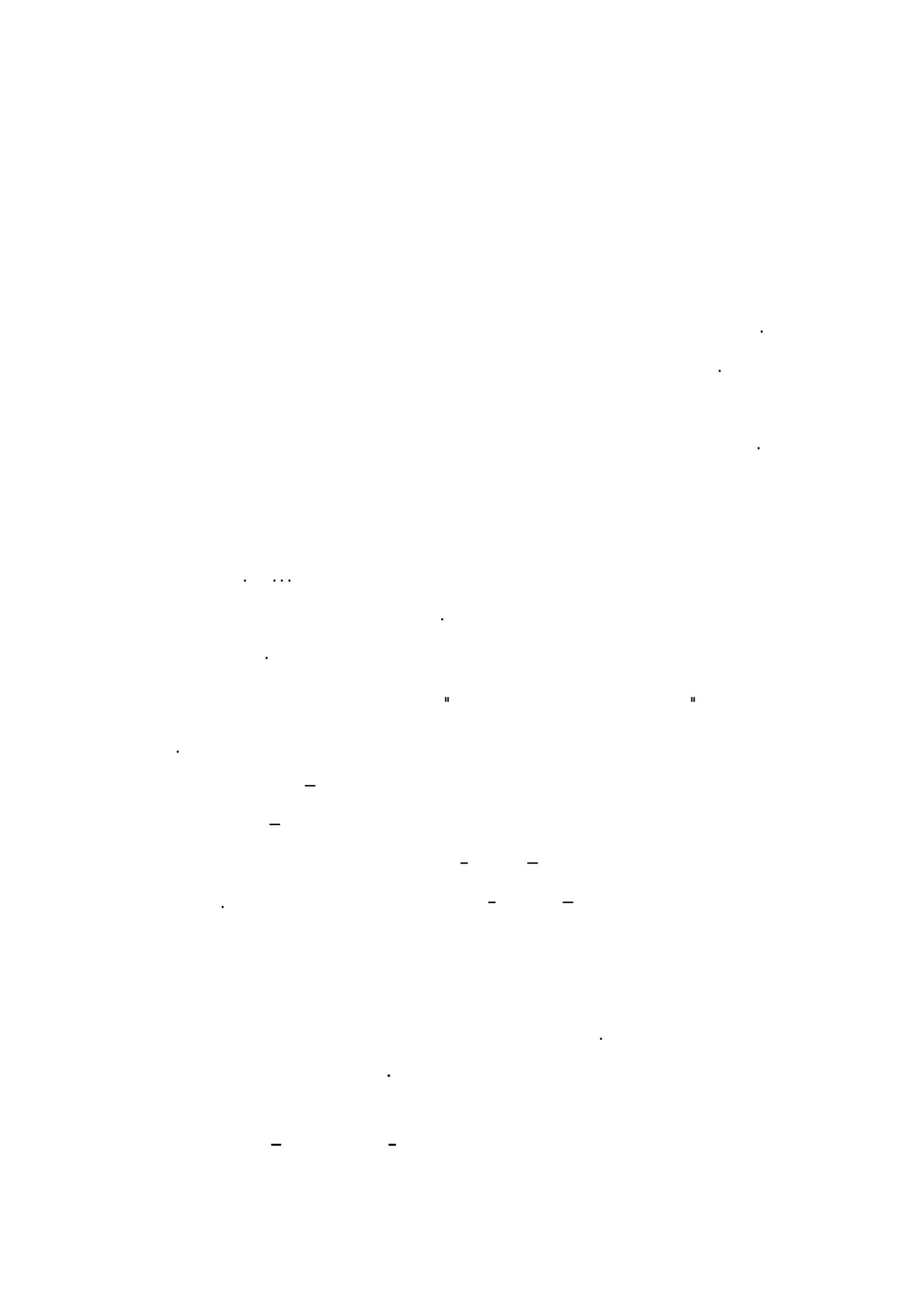
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رقم الإيداع بدار الكتب المصرية: /

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النتائج التعليمية المستهدفة

Statistica

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Statistics

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Descriptive Statistics

Statistical Inference

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Qualitative () Quantitative

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D	B	E	C	D	B	D	C	E	A
B	E	C	D	B	D	D	A	E	C
C	D	A	C	E	D	C	C	D	B
D	E	D	D	A	D	D	C	D	C
D	A	B	D	B	D	C	D	C	E
D	B	C	C	E	D	C	C	D	A

...(B) (A)

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51	95	70	74	73	90	71	74	90	67
91	72	83	89	50	80	72	84	85	69
62	82	87	76	91	76	87	75	78	79
71	96	81	88	64	82	73	57	86	70
80	81	75	85	74	90	83	66	77	91

70

...

90

80

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(Quantitative Data)

(Qualitative Data)

: (Qualitative Data)

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: (Quantitative Data)

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(Frequency Distribution)

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		()
A		6
B		8
C		16
D		22
E		8
		60

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	()
A	6
B	8
C	16
D	22
E	8
	60

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() : (-)

	A	B	C	D	E	
	6	8	16	22	8	60

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$$R = 97 - 50 = 47$$

$$: R - 5 = - - -$$

: L

$$L = 47 / 5 = 9.4 \sim 10$$

$$50 \quad () \quad . 50 + 10 = 60 \quad : - - -$$

$$69 \quad 59$$

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() : (-)

		()
50-59		3
60-69		5
70-79		18
80-89		16
90-99		8
		50

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	()
50-59	3
60-69	5
70-79	18
80-89	16
90-99	8
	50

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	50-59	60-69	70-79	80-89	90-99	
	3	5	18	16	8	50

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.Relative Frequency Table

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.Percentage Frequency Table

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50-59	0.06
60-69	0.10
70-79	0.36
80-89	0.32
90-99	0.16
	1.00

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50-59	6
60-69	10
70-79	36
80-89	32
90-99	16
	100

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	()
49.5-59.5	3
59.5-69.5	5
69.5-79.5	18
79.5-89.5	16
89.5-99.5	8
	50

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50-59	49.5-59.5	54.5	3	0.06	6
60-69	59.5-69.5	64.5	5	0.10	10
70-79	69.5-79.5	74.5	18	0.36	36
80-89	79.5-89.5	84.5	16	0.32	32
90-99	89.5-99.5	94.5	8	0.16	16
			50	1.00	100

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Cumulative Frequency "Less Than"

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< 49.5	0
< 59.5	3
< 69.5	8
< 79.5	26
< 89.5	42
< 99.5	50

Cumulative Frequency "or More"

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= + :

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>49.5	50
>59.5	47
>69.5	42
>79.5	24
>89.5	8
>99.5	0

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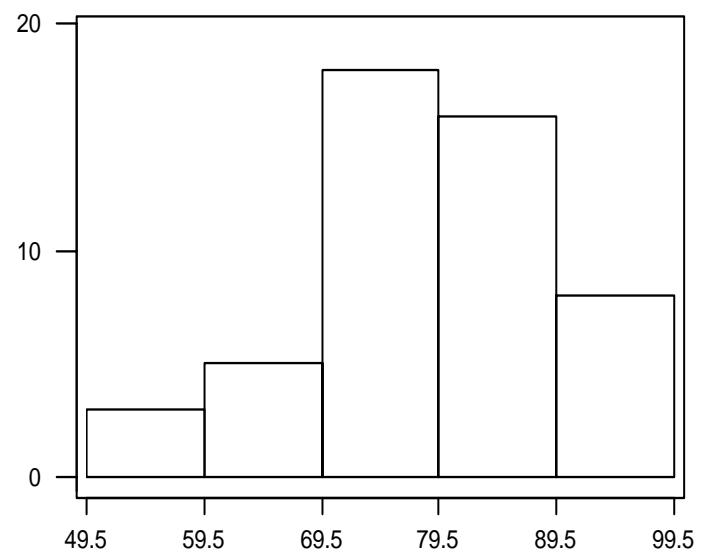
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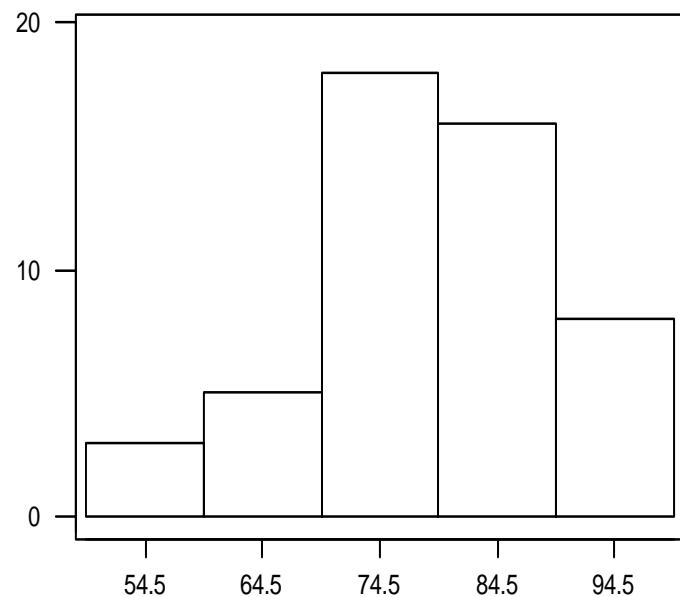
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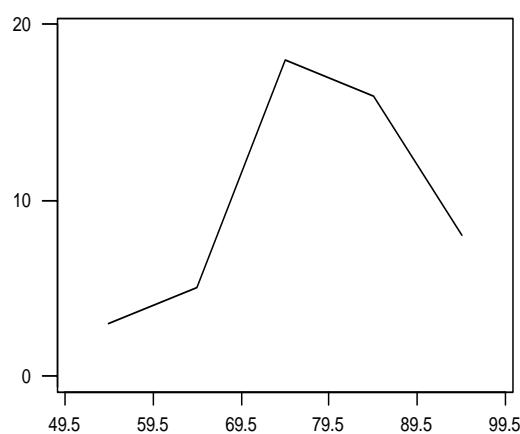
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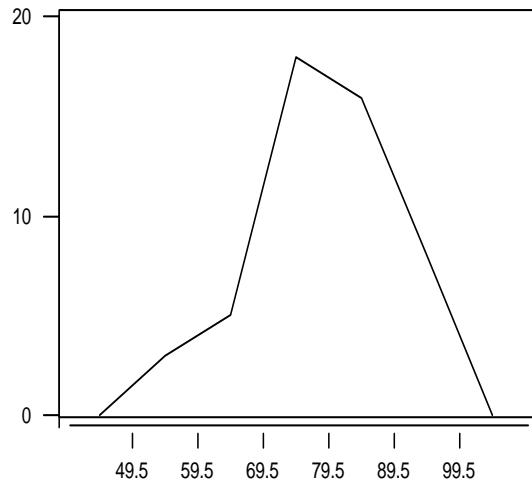
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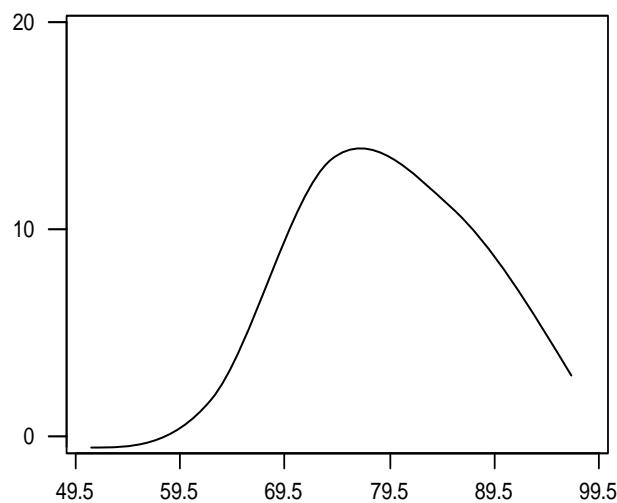
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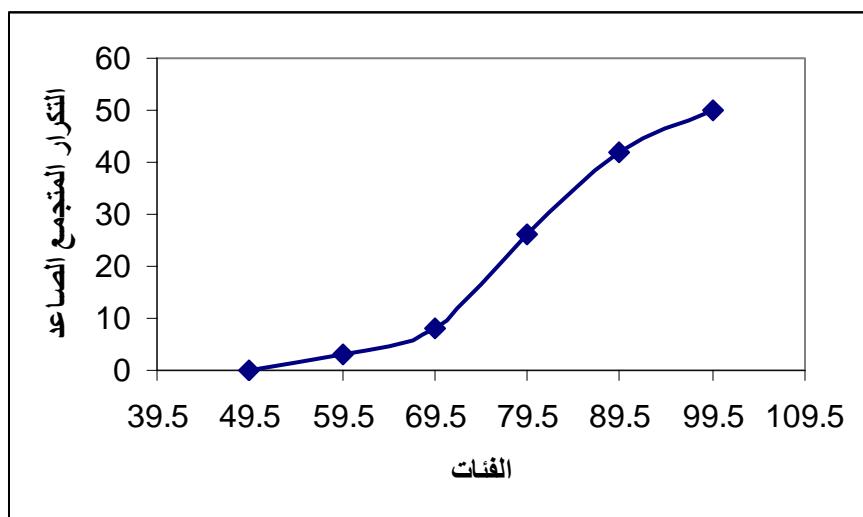
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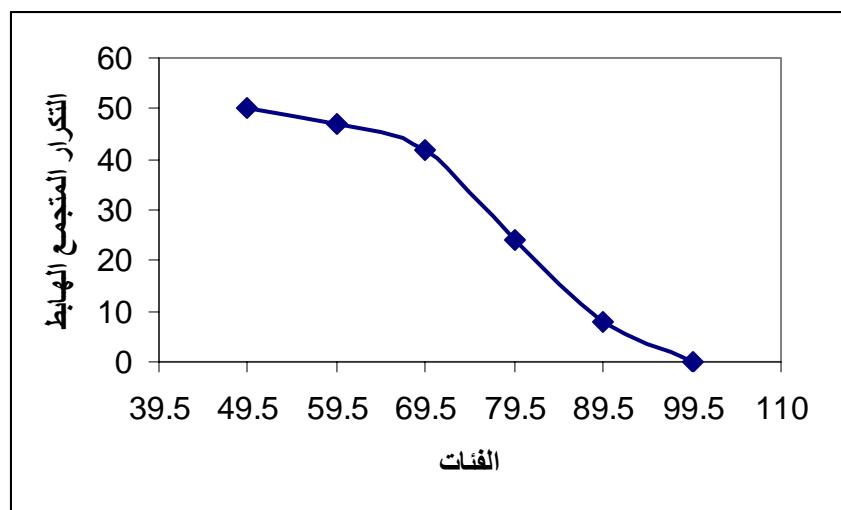
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	50-69	70-79	80-89	90-99	
	8	18	16	8	50

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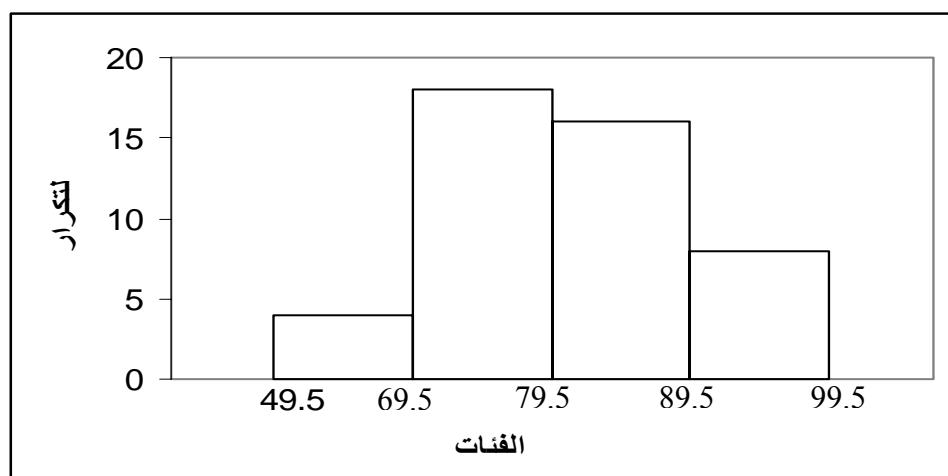
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	49.5-69.5	69.5-79.5	79.5-89.5	89.5-99.5
	8	18	16	8
	4	18	16	8

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132	125	117	124	103	117	110	127	96	129
130	122	118	114	103	119	106	125	114	100
125	128	106	111	115	123	119	114	117	143
136	92	115	118	121	137	139	120	104	125
119	115	101	129	87	108	110	133	135	126
127	103	110	126	118	82	104	137	120	95
146	126	119	105	132	126	118	100	113	119
106	125	117	102	146	129	124	113	95	148

80-89, 90-99, 100-109, ..., 140-149

44	98	40	60	66	71	82	64	72	68
55	69	77	78	88	60	65	68	79	69
62	64	71	66	61	75	83	70	55	62
57	72	61	62	74	62	67	66	60	50

40-49, 50-59, ..., 90-99

0-59	E
60-69	D
70-79	C
80-89	B
90-99	A

	50-59	60-69	70-79	80-89	90-99	100-119	120-129
	8	10	16	15	10	8	3

	155-	158-	161-	164-	167-	170-	173-	176-
	4	10	77	235	368	220	80	6

$$(\quad) \quad : /$$

$$(\quad) \quad$$

$$\vdots \quad \vdots$$

$$(\quad)$$

$$\Sigma$$

$$(x_1, x_2, \dots, x_n)$$

$$\vdots$$

$$\sum_{i=1}^n x_i = x_1 + x_2 + \dots + x_n$$

()

: /

$\sum x$

y

x

(y_1, y_2, \dots, y_n)

$$(i) \sum_{i=1}^n (x_i \pm y_i) = \sum_{i=1}^n x_i \pm \sum_{i=1}^n y_i$$

$$(ii) \sum_{i=1}^n c = nc$$

$$(iii) \sum_{i=1}^n cx_i = c \sum_{i=1}^n x_i$$

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(x_1, x_2, \dots, x_n) x

: \bar{x}

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n}$$

$$= \frac{1}{n} \sum_{i=1}^n x_i$$

: ()

60, 72, 40, 80, 63

$$\begin{aligned} \sum_{i=1}^5 x_i &= 60 + 72 + 40 + 80 + 63 \\ &= 315 \end{aligned}$$

()

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$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i = \frac{1}{5} (315) = 63$$

$$\sum_{i=1}^5 x_i = 63 + 63 + 63 + 63 + 63 = 315$$

$$f_1, f_2, \dots, f_k \quad (x_1, x_2, \dots, x_k) \quad k$$

$$\bar{x} = \frac{f_1 x_1 + f_2 x_2 + \dots + f_k x_k}{f_1 + f_2 + \dots + f_k}$$

$$= \frac{\sum_{i=1}^k f_i x_i}{\sum_{i=1}^k f_i}$$

$$= \frac{1}{n} \sum_{i=1}^k f_i x_i$$

$$: \quad n = \sum_{i=1}^k f_i$$

: \bar{x} : ()

	5-6	7-8	9-10	11-12	13-14
	2	5	8	4	1

: :

	(x)	(f)	$x f$
5-6	5.5	2	11
7-8	7.5	5	37.5
9-10	9.5	8	76
11-12	11.5	4	46
13-14	13.5	1	13.5
		20	184

$$\bar{x} = \frac{1}{n} \sum_{i=1}^k f_i x_i = \frac{1}{20} (184) = 9.2$$

() : /

$$(d_1, d_2, \dots, d_n) \quad (x_1, x_2, \dots, x_n)$$

$$d_i = x_i - \bar{x}, \quad i = 1, 2, \dots, n$$

$$\sum_{i=1}^n (x_i - \bar{x}) = \sum_{i=1}^n d_i = 0$$

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \Rightarrow n \bar{x} = \sum_{i=1}^n x_i \quad \dots \dots (1)$$

$$\because \sum_{i=1}^n (x_i - \bar{x}) = \sum_{i=1}^n x_i - \sum_{i=1}^n \bar{x} = \sum_{i=1}^n x_i - n \bar{x} = 0 \quad \dots \dots (2)$$

$$(x_1, x_2, \dots, x_n)$$

x

$$: \quad (d_1, d_2, \dots, d_n) \quad b \quad x$$

$$d_i = x_i \pm b, \quad i = 1, 2, \dots, n$$

$$\bar{x} = \bar{d} \mp b$$

$$\because d_i = x_i \pm b, \quad i = 1, 2, \dots, n$$

$$\Rightarrow \sum_{i=1}^n d_i = \sum_{i=1}^n (x_i \pm b) = \sum_{i=1}^n x_i \pm n b$$

$$\Rightarrow \frac{1}{n} \sum_{i=1}^n d_i = \frac{1}{n} \sum_{i=1}^n x_i \pm b$$

$$\Rightarrow \bar{d} = \bar{x} \pm b$$

$$\Rightarrow \bar{x} = \bar{d} \mp b$$

$$: \quad \sum_{i=1}^5 d_i \quad b = 50 \quad () \quad : ()$$

$$\sum_{i=1}^5 d_i = (60 - 50) + (72 - 50) + (40 - 50) + (80 - 50) + (63 - 50)$$

$$= 10 + 22 - 10 + 30 + 13 = 65$$

() : /

$$\bar{d} = \frac{1}{5} (65) = 13$$

$$\bar{x} = \bar{d} + b = 13 + 50 = 63$$

.()

$$(x_1, x_2, \dots, x_n) : \underline{\hspace{2cm}}$$

$$: a \bar{x} \quad a$$

$$\overline{(ax)} = a \bar{x}$$

:

$$\overline{(ax \pm b)} = a \bar{x} \pm b$$

$$(x_1, x_2, \dots, x_n) : \underline{\hspace{2cm}}$$

c

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$$\sum_{i=1}^n (x_i - c)^2 \geq \sum_{i=1}^n (x_i - \bar{x})^2, \bar{x} \neq c$$

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60, 72, 40, 80, 63

() : /

40, 60, 63, 72, 80

) 63 ()

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72, 60, 72, 40, 80, 63

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40, 60, 63, 72, 72, 80

Med = (63+72) / 2 = 67.5

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n ($\frac{n}{2}$) -

$\frac{n}{2}$ f_2 f_1

A

: L

Med = $A + \frac{(\frac{n}{2} - f_1)}{f_2 - f_1} L$

f_2 f_1

: () : ()

: :

< 4.5	0
< 6.5	2
< 8.5	7
< 10.5	15
< 12.5	19
< 14.5	20

$$15, 7 \qquad \qquad 10 \qquad \qquad \left(\frac{20}{2} = 10\right) \qquad \qquad \frac{n}{2}$$

10

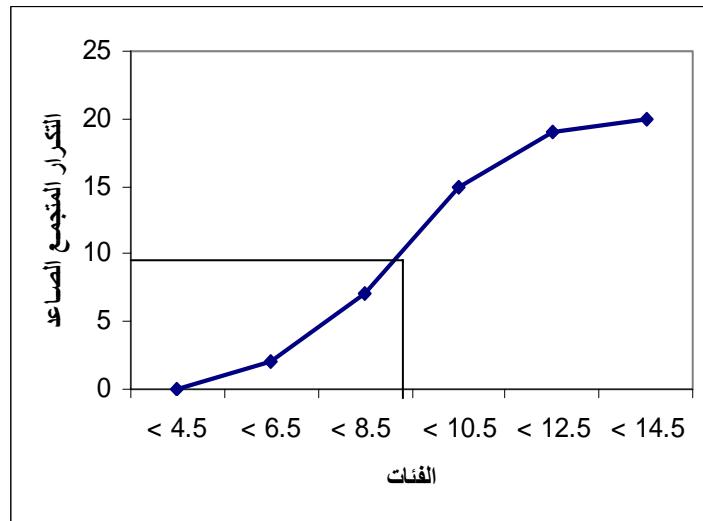
$$A = 8.5, \quad f_1 = 7, \quad f_2 = 15, \quad L = 10.5 - 8.5 = 2$$

$$\text{Med} = 8.5 + \frac{10-7}{15-7} \cdot 2 = 9.25,$$

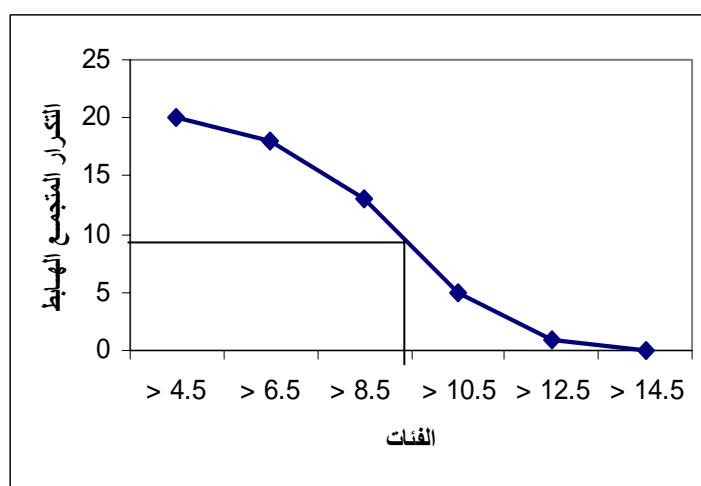
$$\frac{n}{2}$$

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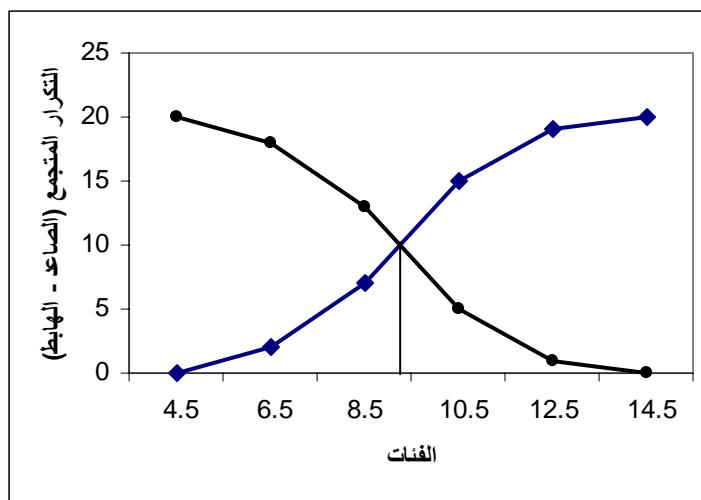
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$$\sum_{i=1}^n |x_i - Med| \leq \sum_{i=1}^n |x_i - a|$$

$a \neq Med$:

()

: ()

2, 6, 9, 4, 6, 10, 6

6

: ()

4, 3, 7, 9, 4, 4, 7, 4

4

: ()

5, 7, 5, 7, 8, 9, 7, 5, 10

() : /

7 5 :
.5,7

: ()
4, 9, 8, 12, 11, 7, 15

: :

()

: :

: :

. f_2 f_1 f -

. f A -

L -

: :

$$Mod = A + \frac{f - f_1}{2f - f_1 - f_2} \cdot L$$

. () : ()

: :

	5 - 6	7 - 8	9 - 10	11 - 12	13 - 14
	2	5	8	4	1

: :

$$f = 8, \quad f_1 = 5, \quad f_2 = 4, \quad A = 8.5$$
$$L = 10.5 - 8.5 = 2$$

: :

$$Mod = 8.5 + \frac{8 - 5}{16 - 5 - 4} \cdot 2$$

$$Mod = 9.36$$

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x_1, x_2, \dots, x_n

G.M.

$\sqrt[n]{x_1 x_2 \dots x_n}$ G.M. = :

Log G.M. = $\frac{1}{n} \left(\sum_{i=1}^n \log(x_i) \right)$

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3, 5, 6, 6, 7, 10, 12

:

G.M. = $\sqrt[7]{3.5.6.6.7.10.12}$

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$$\begin{aligned}\text{Log G.M.} &= \frac{1}{7}(\text{Log } 3 + \text{Log } 5 + \text{Log } 6 + \text{Log } 6 + \text{Log } 7 + \text{Log } 10 + \text{Log } 12) \\ &= \frac{1}{7}(0.4771 + 0.699 + 0.7782 + 0.7782 + 0.8451 + 1 + 1.0729) \\ &= 0.8081\end{aligned}$$

G.M. = 6.43

$$\bar{x} = \frac{1}{7}(3 + 5 + 6 + 6 + 7 + 10 + 12) = 7$$

G.M.

\bar{x}

()

x_1, x_2, \dots, x_k

k

:

f_1, f_2, \dots, f_k

G.M. = $\sqrt[n]{x_1^{f_1} x_2^{f_2} \dots x_n^{f_n}}$

$$n = \sum_{i=1}^k f_i$$

-

H

.

x_1, x_2, \dots, x_n

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$$H = \frac{1}{\frac{1}{n} \sum_{i=1}^n \frac{1}{x_i}} = \frac{n}{\sum_{i=1}^n \frac{1}{x_i}}$$

:

$$\frac{1}{H} = \frac{1}{n} \sum_{i=1}^n \frac{1}{x_i}$$

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3,5,6,6,7,10,12

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$$\frac{1}{H} = \frac{1}{7} \left(\frac{1}{3} + \frac{1}{5} + \frac{1}{6} + \frac{1}{6} + \frac{1}{10} + \frac{1}{12} \right)$$

$$\frac{1}{H} = \frac{1}{7} \left(\frac{140 + 84 + 70 + 70 + 60 + 42 + 35}{420} \right)$$

$$\frac{1}{H} = \frac{501}{2940} \Rightarrow H = 5.87$$

> >

() k

f_1, f_2, \dots, f_k

x_1, x_2, \dots, x_k

k

:

$$\frac{1}{H} = \frac{1}{n} \sum_{i=1}^n \frac{f_i}{x_i} = \frac{1}{n} \left(\frac{f_1}{x_1} + \frac{f_2}{x_2} + \dots + \frac{f_k}{x_k} \right)$$

$$n = \sum_{i=1}^k f_i :$$

-

$Q_1 \quad Q_3 \quad Q_2 \quad Q_1$

$Q_3 \quad (\quad)$

Q_2

$D_9 \quad \dots \quad D_2 \quad D_1$

D_2

D_1

$P_1 \quad P_{99} \quad \dots \quad P_2 \quad P_1$

P_2

-

()

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$$\frac{2n}{4} \quad \frac{n}{4} \quad \frac{n}{2} \quad () \quad \frac{2n}{100} \quad \frac{n}{100} \quad \frac{n}{2} \quad .$$

· () : ()

< 4.5	0
< 6.5	2
D ₂	
< 8.5	7
< 10.5	15
P ₉₀	
< 12.5	19
< 14.5	20

$$D_2 = A + \frac{\left(\frac{2n}{10} - f_1\right)}{f_2 - f_1} L$$

$$f_2 \qquad \qquad f_1 \qquad \qquad n \qquad \qquad A$$

$$D_2 = 6.5 + \frac{4-2}{7-2} \cdot 2 = 7.3$$

⋮ P_{90}

$$P_{90} = A' + \frac{\left(\frac{90n}{100} - f_1'\right)}{f_2' - f_1'} L$$

$$P_{90} = 10.5 + \frac{18-15}{19-15} \cdot 2 = 12$$

x	1	2	4	-2	5	
y	-1	1	5	-7	7	

$$\text{i) } \sum x, \sum y, \sum xy, \sum x^2, \sum y^2, \sum x^3, \sum y^3, \sum xy^2$$

$$\text{ii) } (\sum x)^2, ((\sum x) + 2)^2, \sum (x - 3y)^2, \sum x(x + 4)$$

$$\text{(i) } \sum \left(\frac{y}{x}\right) = \frac{\sum y}{\sum x}, \quad \text{ii) } \sum xy = \sum x \sum y$$

$$\text{(iii) } \sum (x - c)(x + c) = \sum x^2 - nc^2, \quad \text{(iv) } (\sum x)^2 = \sum x^2$$

6,6,9,8,6,10,9,9,8,7,8,6,7,8,8,11,10,11,8,8

2	8
3	10
5	20
7	20
8	6
9	6

	40-49	50-59	60-69	70-79	80-89	90-99
	6	10	17	17	6	4

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	20-29	30-39	40-49	50-59	60-69	70-79
	9	12	15	8	4	2

0, 2, 4, 6

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: Z Y X

X	59,61,62,58,60
Y	50,60,66,54,70
Z	39,65,46,78,72

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Measures of Dispersion

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- = R

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R : ()

82, 40, 62, 70, 30, 80

= = :

$$R = 82 - 30 = 52$$

R : ()

	40-49	50-59	60-69	70-79	80-89	90-99
	2	9	15	11	2	1

$$R = 94.5 - 44.5 = 50$$

$$R = 99.5 - 39.5 = 60$$

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n

$Q_3 \quad Q_1$

Q

$$Q = \frac{Q_3 - Q_1}{2}, \quad (1)$$

$$Q_2 \quad \frac{n}{2} \quad Q_2$$

Q

Q_1

Q_3

()

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67,65,69,58,55,71,72,70

: :

55,58,65,67,69,70,71,72

$$Q_1 = \frac{58+65}{2} = 61.5,$$

$$Q_3 = \frac{70+71}{2} = 70.5$$

$$Q = \frac{Q_3 - Q_1}{2} = 4.5$$

: ()

59,67,65,69,58,55,70,72,74

: :

55,58,59,65,67,69,70,72,74

$$Q_1 = 59,$$

$$Q_3 = 70$$

$$Q = \frac{Q_3 - Q_1}{2} = 5.5$$

$\frac{n}{2}$ $\frac{n}{4}$ Q_1

$\frac{n}{2}$ $\frac{3n}{4}$ Q_3

: Q_3 Q_1 ()

$$Q_1 = A_1 + \frac{\left(\frac{n}{4} - f_1\right)}{f_2 - f_1} L, \quad (2)$$

$$Q_3 = A_2 + \frac{\left(\frac{3n}{4} - f_1\right)}{f'_2 - f'_1} L, \quad (3)$$

. () : ()

: :

< 39.5	0
< 49.5	2
< 59.5	11
< 69.5	26
< 79.5	37
< 89.5	39
< 99.5	40

$$n = 40, \quad \frac{n}{4} = 10, \quad \frac{3n}{4} = 30, \quad L = 10$$

$$Q_1 = 49.5 + \left(\frac{10-2}{11-2} \right) 10$$

$$Q_1 = 49.5 + 8.89 = 58.39$$

$$Q_3 = 69.5 + \left(\frac{30-26}{37-26} \right) 10$$

$$Q_3 = 69.5 + 3.64 = 73.14$$

$$Q = \frac{Q_3 - Q_1}{2} = 7.38$$

\bar{x}

M.D.

$$M.D. = \frac{1}{n} \sum_{i=1}^n |x_i - \bar{x}|, \quad (4)$$

() .()

:

$$\text{M.D.} = \frac{\sum_{i=1}^n f_i |x_i - \bar{x}|}{n}, \quad (5)$$

n

6,5,7,7,8,9,9,5

: :

x	x - \bar{x}	x - \bar{x}
6	-1	1
5	-2	2
7	0	0
7	0	0
8	1	1
9	2	2
9	2	2
5	-2	2
56	0	10

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{56}{8} = 7$$

$$\text{M.D.} = \frac{10}{8} = 1.25,$$

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Classes	x	f	$f \cdot x$	$x - \bar{x}$	$ x - \bar{x} $	$ x - \bar{x} f$
40-49	44.5	2	89	-21.25	21.25	42.5
50-59	54.5	9	490.5	-11.25	11.25	101.25
60-69	64.5	15	967.5	-1.25	1.25	18.75
70-79	74.5	11	819.5	8.75	8.75	96.75
80-89	84.5	2	169	18.75	18.75	37.5
90-99	94.5	1	94.5	28.75	28.75	28.75
		40	2630			325

$$\bar{x} = \frac{\sum f x}{n}$$

$$\bar{x} = \frac{2630}{40} = 65.75$$

$$M.D. = \frac{325}{40} = 8.125,$$

:

()

$$\sigma^2$$

$$\sigma$$

:

 N

$$\bar{X}$$

$$X_1, X_2, \dots, X_N$$

:

$$(X_1 - \bar{X})^2, (X_2 - \bar{X})^2, \dots, (X_N - \bar{X})^2$$

()

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: σ^2

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^N (X_i - \bar{X})^2, \quad (6)$$

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N (X_i - \bar{X})^2}, \quad (7)$$

$$\begin{array}{cccccc} & & & & & \\ & () & & & & \\ n & & & .() & & \\ & S^2 & & S & & \\ & & & & & \\ & & & & & : \\ & & & & & (n-1) \end{array}$$

$$S^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2, \quad (8)$$

$$S = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2}, \quad (9)$$

$$\begin{array}{ccccc} & & & & S \\ \sigma^2 & S^2 & () & & \\ & & & & \end{array}$$

$$\begin{array}{ccccc} & & & & : () \\ & & & & \\ & & & & 8,9,7,6,5 \\ & & & & \\ & & & & : \end{array}$$

x	$x - \bar{x}$	$(x - \bar{x})^2$
8	1	1
9	2	4
7	0	0
6	-1	1
5	-2	4
35	0	10

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{35}{5} = 7$$

$$S^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2,$$

()

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$$S^2 = \frac{1}{5-1}(10) = 2.5$$

$$S = \sqrt{2.5} = 1.581,$$

: ()

$$S^2 = \frac{1}{n-1} \left(\sum_{i=1}^n x_i^2 - \frac{\left(\sum_{i=1}^n x_i \right)^2}{n} \right), \quad (10)$$

:

$$S^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$$

$$S^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i^2 - 2x_i \bar{x} + \bar{x}^2)$$

$$S^2 = \frac{1}{n-1} \left(\sum_{i=1}^n x_i^2 - 2\bar{x} \sum_{i=1}^n x_i + n\bar{x}^2 \right)$$

$$S^2 = \frac{1}{n-1} \left(\sum_{i=1}^n x_i^2 - n\bar{x}^2 \right)$$

$$S^2 = \frac{1}{n-1} \left(\sum_{i=1}^n x_i^2 - \frac{\left(\sum_{i=1}^n x_i \right)^2}{n} \right)$$

$$\cdot \quad \sum x^2 \quad \sum x \quad ()$$

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x	x^2
8	64
9	81
7	49
6	36
5	25
35	255

$$S^2 = \frac{1}{n-1} \left(\sum_{i=1}^n x_i^2 - \frac{\left(\sum_{i=1}^n x_i \right)^2}{n} \right)$$

$$S^2 = \frac{1}{4} \left(255 - \frac{(35)^2}{5} \right) = 2.5$$

$$S = \sqrt{2.5} = 1.581,$$

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c

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$$\begin{array}{lll} \vdots & d_1, d_2, \dots, d_n & x_1, x_2, \dots, x_n \\ & & d_n = x_n + c, \dots, d_2 = x_2 + c, d_1 = x_1 + c \end{array}$$

$$\therefore S^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$$

$$S^2 = \frac{1}{n-1} \left(\sum_{i=1}^n [(d_i \pm c) - (\bar{d} \pm c)]^2 \right) = \frac{1}{n-1} \sum_{i=1}^n (d_i - \bar{d})^2, \quad (11)$$

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$$\begin{array}{lll} \cdot & c & () \\ & & : () \end{array}$$

:

x	$d = x - 5$	d^2
8	3	9
9	4	16
7	2	4
6	1	1
5	0	0
	10	30

$$S^2 = \frac{1}{n-1} \left(\sum_{i=1}^n d_i^2 - \frac{(\sum_{i=1}^n d_i)^2}{n} \right)$$

$$S^2 = \frac{1}{4} (30 - \frac{(10)^2}{5}) = 2.5$$

$$S = \sqrt{2.5} = 1.581,$$

$$\begin{array}{ll} \cdot & () \end{array}$$

:

$$\begin{array}{ll} c & x_1, x_2, \dots, x_n \end{array}$$

$$\vdots \quad d_1, d_2, \dots, d_n$$

$$d_1 = cx_1, d_2 = cx_2, \dots, d_n = cx_n$$

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$$(\quad \quad) : /$$

$$\begin{aligned}
 S_x^2 &= \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2 \\
 S_x^2 &= \frac{1}{n-1} \sum_{i=1}^n \left(\frac{d_i}{c} - \frac{\bar{d}}{c} \right)^2 = \frac{1}{n-1} \frac{1}{c^2} \sum_{i=1}^n (d_i - \bar{d})^2 \\
 S_x^2 &= \frac{1}{c^2} S_d^2, \Rightarrow S_x = \frac{1}{c} S_d,
 \end{aligned} \tag{12}$$

$$(\quad)$$

$$S_x = c S_d, \tag{13}$$

$$\begin{aligned}
 \sum (x - a)^2 &= \sum (x + \bar{x} - \bar{x} - a)^2 \\
 &= \sum [(x - \bar{x}) + (\bar{x} - a)]^2 \\
 &= \sum (x - \bar{x})^2 + n(\bar{x} - a)^2 + 2(\bar{x} - a) \sum (x - \bar{x}) \\
 &= \sum (x - \bar{x})^2 + n(\bar{x} - a)^2 \\
 &\quad \vdots \qquad \qquad \qquad n(\bar{x} - a)^2 \\
 S_2^2 &\quad S_1^2 \qquad \qquad n_2 \quad n_1 \qquad \qquad \vdots \qquad \qquad \sum (x - a)^2 < \sum (x - \bar{x})^2 \\
 &\quad \vdots \qquad \qquad \qquad \qquad \qquad \qquad \qquad \bar{x}
 \end{aligned}$$

$$S^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 1}$$

$$\begin{aligned}
 x_1, x_2, \dots, x_{n_1}, \quad y_1, y_2, \dots, y_{n_2} \\
 S_1^2 &= \frac{1}{n_1 - 1} \sum_{i=1}^{n_1} (x_i - \bar{x})^2 \\
 S_2^2 &= \frac{1}{n_2 - 1} \sum_{i=1}^{n_2} (y_i - \bar{x})^2
 \end{aligned}$$

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$$(n_1 - 1) S_1^2 = \sum_{i=1}^{n_1} (x_i - \bar{x})^2$$

$$(n_2 - 1) S_2^2 = \sum_{i=1}^{n_2} (y_i - \bar{y})^2$$

$$(n_1 - 1) S_1^2 + (n_2 - 1) S_2^2 = \sum_{i=1}^{n_1} (x_i - \bar{x})^2 + \sum_{i=1}^{n_2} (y_i - \bar{y})^2 = \sum_{i=1}^{n_1+n_2} (z_i - \bar{x})^2$$

$$\therefore S^2 = \frac{(n_1 - 1) S_1^2 + (n_2 - 1) S_2^2}{n_1 + n_2 - 1}$$

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f_1, f_2, \dots, f_k

x_1, x_2, \dots, x_k

k

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$$S^2 = \frac{1}{n-1} \sum_{i=1}^n f_i (x_i - \bar{x})^2, \quad S = \sqrt{\frac{1}{n-1} \sum_{i=1}^n f_i (x_i - \bar{x})^2}, \quad (14)$$

$$S^2 = \frac{1}{n-1} \left(\sum_{i=1}^n f_i x_i^2 - \frac{\left(\sum_{i=1}^n f_i x_i \right)^2}{n} \right), \quad (15)$$

$$S^2 = \frac{1}{n-1} \left(\sum_{i=1}^n f_i d_i^2 - \frac{\left(\sum_{i=1}^n f_i d_i \right)^2}{n} \right), \quad (16)$$

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Classes	x	f	xf	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})^2 f$
40-49	44.5	2	89	-21.25	451.56	903.13
50-59	55.5	9	490.5	-11.25	126.56	1139.06
60-69	65.5	15	967.5	-1.25	1.56	23.44
70-79	75.5	11	819.5	8.75	76.56	842.19
80-89	85.5	2	169	18.75	351.56	703.13
90-99	95.5	1	94.5	28.75	826.56	826.56
Total		40	2630			4437.5

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$$\bar{x} = \frac{1}{n} \sum_{i=1}^k f_i x_i = \frac{1}{40} (2630) = 65.75$$

$$S^2 = \frac{1}{n-1} \sum_{i=1}^n f_i (x_i - \bar{x})^2 = \frac{1}{40-1} (4437.5) = 113.78$$

$$S = 10.67,$$

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Classes	x	f	xf	$x^2 f$
40-49	44.5	2	89	3960.5
50-59	55.5	9	490.5	26732.25
60-69	65.5	15	967.5	62403.75
70-79	75.5	11	819.5	61052.75
80-89	85.5	2	169	12280.5
90-99	95.5	1	94.5	8930.25
Total		40	2630	177360

$$S^2 = \frac{1}{n-1} \left(\sum_{i=1}^n f_i x_i^2 - \frac{\left(\sum_{i=1}^n f_i x_i \right)^2}{n} \right) = \frac{1}{40-1} (177360 - 172922.5) = 113.78$$

$$S = 10.67, ()$$

$c = 64.5$ () () -

: ()

Classes	x	f	$d = x - 64.5$	df	$d^2 f$
40-49	44.5	2	-20	-40	800
50-59	55.5	9	-10	-90	900
60-69	65.5	15	0	0	0
70-79	75.5	11	10	110	1100
80-89	85.5	2	20	40	800
90-99	95.5	1	30	30	900
Total		40		50	4500

$$S^2 = \frac{1}{n-1} \left(\sum_{i=1}^n f_i d_i^2 - \frac{\left(\sum_{i=1}^n f_i d_i \right)^2}{n} \right) = \frac{1}{40-1} (4500 - 62.5) = 113.78$$

$$S = 10.67,$$

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Classes	x	f	$d = x - 64.5$	$d/10 = d'$	$d'f$	$d'^2 f$
40-49	44.5	2	-20	-2	-4	8
50-59	55.5	9	-10	-1	-9	9
60-69	65.5	15	0	0	0	0
70-79	75.5	11	10	1	11	11
80-89	85.5	2	20	2	4	8
90-99	95.5	1	30	3	3	9
Total		40			5	45

$$S_{d'}^2 = \frac{1}{n-1} \left(\sum_{i=1}^n f_i d_i'^2 - \frac{\left(\sum_{i=1}^n f_i d_i' \right)^2}{n} \right) = \frac{1}{40-1} (45 - 0.625) = 1.1378$$

$$S_{d'} = 1.067, \quad S_x = 10 \quad S_{d'} = 10.67 \quad ()$$

$$\bar{x} \quad x_1, x_2, \dots, x_n \quad X \\ \vdots \quad \quad \quad Z \quad s$$

$$Z_i = \frac{x_i - \bar{x}}{s}, \quad i = 1, 2, \dots, n \\ " \quad \quad \quad Z_i \\ .(\quad) "$$

:()

$$: \quad \quad \quad Z_1 \quad : \\ Z_1 = \frac{82 - 75}{10} = 0.7 \\ : \quad \quad \quad Z_2 \\ Z_2 = \frac{89 - 81}{16} = 0.5$$

$$1- r = \frac{3(\bar{x} - Med)}{s}, \quad (17)$$

$$2- r = \frac{\bar{x} - Mod}{s}, \quad (18)$$

$$3- r = \frac{m_3^2}{s^3}, \quad (19)$$

$$m_3 = \frac{\sum (x - \bar{x})^3}{n},$$

$$m_3 = \frac{\sum f(x - \bar{x})^3}{n},$$

$$(\quad) \quad : (\quad)$$

$$\bar{x} = 65.75, \quad Med = 65.41, \quad Mod = 65.5, \quad s = 10.67$$

$$r = \frac{3(\bar{x} - Med)}{s} = \frac{3(65.75 - 65.41)}{10.67} = 0.069, \quad (1)$$

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$$r = \frac{\bar{x} - Mod}{s} = \frac{65.75 - 65.5}{10.67} = 0.025, \quad (2)$$

-

: k

$$k = \frac{m_4}{s^4} - 3$$

:

$$m_4 = \frac{\sum (x - \bar{x})^4}{n},$$

$$m_4 = \frac{\sum f(x - \bar{x})^4}{n},$$

: k

-

6,3,5,5,9,4,6,7,1,2,4,8

	58-60	61-63	64-66	67-69	70-72	73-75
	2	7	14	15	8	4

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	< 10	10-14	15-19	20-24	25-29	30≤
	5	20	35	19	13	8

2, 5, 9, 4, 3, 6

70, 70, 70, 70, 70, 70, 70

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...

...

x (x, y) y

(Pearson) (x, y)
(Spearman)

y x

(x_1, y_1), (x_2, y_2), ..., (x_n, y_n)

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$$r = \frac{\sum xy - \frac{(\sum x)(\sum y)}{n}}{\sqrt{(\sum x^2 - \frac{(\sum x)^2}{n})(\sum y^2 - \frac{(\sum y)^2}{n})}}, \quad (1)$$

(r)

(-)

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x	13	9	19	15	11	8	16	11
y	15	7	17	15	10	9	14	10

y x = :

x	y	- 10 x = x	-10 y = y	y x	x^2	y^2
13	15	3	5	15	9	25
9	7	-1	-3	3	1	9
19	17	9	7	63	81	49
15	15	5	5	25	25	25
11	10	1	0	0	1	0
8	9	-2	-1	2	4	1
16	14	6	4	24	36	16
11	10	1	0	0	1	0
		22	17	132	158	125

$$r = \frac{\sum xy - \frac{(\sum x)(\sum y)}{n}}{\sqrt{(\sum x^2 - \frac{(\sum x)^2}{n})(\sum y^2 - \frac{(\sum y)^2}{n})}}$$

$$r = \frac{132 - \frac{(22 \times 17)}{8}}{\sqrt{(158 - \frac{(22)^2}{8})(125 - \frac{(17)^2}{8})}}$$

$$r = 0.93$$

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· (x, y) (x, y)

$$r_s = 1 - \frac{6 \sum d^2}{n(n^2 - 1)}, \quad (2)$$

d (x, y) n r_s

· (x, y)

: ()

:

x	A	C	C	C	B	D
y	B	B	D	C	A	E

:

:

x	y	a = x	b = y	d = a - b	d ²
A	B	6	4.5	1.5	2.25
C	B	3	4.5	-1.5	2.25
C	D	3	2	1	1
C	C	3	3	0	0
B	A	5	6	-1	1
D	E	1	1	0	0
					6.5

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$$r_s = 1 - \frac{6 \sum d^2}{n(n^2 - 1)}$$

$$r_s = 1 - \frac{6 \times 6.5}{6(36 - 1)}$$

$$r_s = 1 - 0.186$$

$$r_s = 0.814$$

.....

:

C.C

:

	A	B
	C	D

: C.C

$$C.C = \frac{AD - BC}{AD + BC}, \quad (3)$$

()

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:()

	5	5
	3	4

$$C.C = \frac{AD - BC}{AD + BC}$$

$$C.C = \frac{(5 \times 4) - (3 \times 5)}{(5 \times 4) + (3 \times 5)}$$

$$C.C = 0.14$$

X

C

s

Y

r

X \ Y	Y_1	Y_2	...	Y_s	
X_1	f_{11}	f_{12}	...	f_{1s}	$f_{1.}$
X_2	f_{21}	f_{22}	...	f_{2s}	$f_{2.}$
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
X_r	f_{r1}	f_{r2}	...	f_{rs}	$f_{r.}$
	$f_{.1}$	$f_{.2}$...	$f_{.s}$	$f_{..}$

()

: /

: C

B

$$C = \sqrt{\frac{B-1}{B}}, \quad (4)$$

:

$$B = \frac{(f_{11})^2}{f_{.1}f_{1.}} + \frac{(f_{12})^2}{f_{.2}f_{1.}} + \dots + \frac{(f_{rs})^2}{f_{.s}f_{r.}}$$

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:

X \ Y			
X	6	4	10
	7	2	9
	6	5	11
	19	11	30

C

: B

:

$$B = \frac{6^2}{19 \times 10} + \frac{7^2}{19 \times 9} + \frac{6^2}{19 \times 11} + \frac{4^2}{11 \times 10} + \frac{2^2}{11 \times 9} + \frac{5^2}{11 \times 11}$$

$$B = 1.05$$

: C

$$C = \sqrt{\frac{B-1}{B}}$$

$$C = \sqrt{\frac{1.05-1}{1.05}}$$

$$C = 0.22$$

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(x, y)

(-)

x

y

x

x y

y

$$y = a + bx,$$

(5)

:

(y)

a

$$a = \bar{y} - b\bar{x},$$

(6)

:

x y

b

$$b = \frac{\sum xy - \frac{(\sum x)(\sum y)}{n}}{\sum x^2 - \frac{(\sum x)^2}{n}},$$

(7)

. ()

x

y

:()

: : :

x	y	xy	x^2	y^2
15	13	195	225	169
7	9	63	49	81
17	19	323	289	361
15	15	225	225	225
10	11	110	100	121
9	8	72	81	64
14	16	224	196	256
10	11	110	100	121
97	102	1322	1265	1398

$$b = \frac{\sum xy - \frac{(\sum x)(\sum y)}{n}}{\sum x^2 - \frac{(\sum x)^2}{n}}$$

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$$b = \frac{1322 - \frac{97 \times 102}{8}}{1265 - \frac{(97)^2}{8}} = 0.96$$

$$a = \bar{y} - b\bar{x}$$

$$a = \frac{102}{8} - 0.96 \left(\frac{97}{8} \right)$$

$$a = 1.11$$

: x y

$$y = 1.11 + 0.96x$$

()

: /

:

x	3	2	1	1	5	6	1	4
y	31	44	60	70	18	17	71	29

()

()

x y

y x

x	56	66	42	44	38	27	39	40
y	31	38	27	22	19	25	20	28

x y

	A	B	D	E	C	D	E	B
	A	C	E	D	C	D	E	B

X

X

()

X

X

• • •

()

$$\cdot (p + q = 1)$$

q

10

)

11

X

6

(

n

$$P(X = x) = \binom{n}{x} p^x q^{n-x}, \quad x = 0, 1, 2, \dots, n$$

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$$n = 15, \quad p = 0.8, \quad q = 0.2$$

$$P(X = x) = \binom{15}{x} (0.8)^x (0.2)^{15-x}, \quad x = 0, 1, 2, \dots, 15$$

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$$P(X = 15) = \binom{15}{15} (0.8)^{15} (0.2)^{15-15}$$

$$P(X = 15) = 1 \times 0.035 \times 1$$

$$P(X = 15) = 0.035$$

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$$P(X = 8) = \binom{15}{8} (0.8)^8 (0.2)^{15-8}$$

$$P(X = 8) = 6435 \times 0.1677722 \times 0.0000128$$

$$P(X = 8) = 0.013819$$

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$$P(X = 6) = \binom{15}{6} (0.8)^6 (0.2)^{15-6}$$

$$P(X = 6) = 5005 \times 0.262144 \times 0.000000512$$

$$P(X = 6) = 0.000672$$

:-

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$$P(X = 0) = \binom{15}{0} (0.8)^0 (0.2)^{15-0}$$

$$P(X = 0) = 1 \times 1 \times 0$$

$$P(X = 0) = 0$$

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$$n = 20, \quad p = 0.6, \quad q = 0.4$$

$$P(X = x) = \binom{20}{x} (0.6)^x (0.4)^{20-x}, \quad x = 0, 1, 2, \dots, 20$$

$$P(X = 7) = \binom{20}{7} (0.6)^7 (0.4)^{20-7}$$

$$P(X = 7) = 77520 \times 0.0279936 \times 0.00000671$$

$$P(X = 7) = 0.014563$$

$$P(X = 0) = \binom{20}{0} (0.6)^0 (0.4)^{20-0}$$

$$P(X = 0) = 1 \times 1 \times 0.000$$

$$P(X = 0) = 0$$

$$P(X = 20) = \binom{20}{20} (0.6)^{20} (0.4)^{20-20}$$

$$P(X = 20) = 1 \times 0.00004 \times 1$$

$$P(X = 20) = 0.00004$$

$$P(X = 10) = \binom{20}{10} (0.6)^{10} (0.4)^{20-10}$$

$$P(X = 10) = 184756 \times 0.006046618 \times 0.0001$$

$$P(X = 10) = 0.111715$$

:()

$$n = 3, \quad p = 0.8, \quad q = 0.2$$

$$P(X = x) = \binom{3}{x} (0.8)^x (0.2)^{3-x}, \quad x = 0, 1, 2, 3$$

$$P(X = 3) = \binom{3}{3} (0.8)^3 (0.2)^{3-3}$$

$$P(X = 3) = 1 \times 0.512 \times 1$$

$$P(X = 3) = 0.512$$

$$1 - P(X = 2) = 1 - \binom{3}{2} (0.8)^2 (0.2)^{3-2}$$

$$= 1 - (3 \times 0.64 \times 0.2)$$

$$= 1 - 0.384$$

$$1 - P(X = 2) = 0.616$$

$$P(X = 0) = \binom{3}{0} (0.8)^0 (0.2)^{3-0}$$

$$P(X = 0) = 1 \times 1 \times 0.008$$

$$P(X = 0) = 0.008$$

$$P(X \geq 1) = P(X = 1) + P(X = 2) + P(X = 3)$$

$$P(X \geq 1) = \binom{3}{1} (0.8)^1 (0.2)^{3-1} + \binom{3}{2} (0.8)^2 (0.2)^{3-2} + \binom{3}{3} (0.8)^3 (0.2)^{3-3}$$

$$P(X \geq 1) = 0.096 + 0.384 + 0.512$$

$$P(X \geq 1) = 0.992$$

$$P(X \leq 1) = P(X = 1) + P(X = 0)$$

$$P(X \leq 1) = 0.096 + 0.008$$

$$P(X \leq 1) = 0.104$$

X

$$\lambda \quad x = 0, 1, 2, \dots \quad (\dots) \quad)$$

X

$$P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}, \quad x = 0, 1, 2, \dots$$

:()

$$\lambda = 5 ,$$

$$P(X = x) = \frac{e^{-5} 5^x}{x!}, \quad x = 0, 1, 2, \dots$$

$$1 - P(X = 10) = \frac{e^{-5} 5^{10}}{10!}$$

$$P(X = 10) = 0.018132789$$

$$2 - P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$$

$$P(X < 3) = \frac{e^{-5} 5^0}{0!} + \frac{e^{-5} 5^1}{1!} + \frac{e^{-5} 5^2}{2!}$$

$$P(X < 3) = 0.006737947 + 0.033689735 + 0.084224337$$

$$P(X < 3) = 0.124652$$

$$3 - P(X > 1) = 1 - P(X \leq 1)$$

$$P(X > 1) = 1 - (P(X = 0) + P(X = 1))$$

$$P(X > 1) = 1 - (0.006737947 + 0.033689735)$$

$$P(X > 1) = 1 - 0.040428$$

$$P(X > 1) = 0.959572318$$

$$4 - P(4 \leq X \leq 8) = P(X = 4) + P(X = 5) + P(X = 6) + P(X = 7) + P(X = 8)$$

$$P(4 \leq X \leq 8) = \frac{e^{-5} 5^4}{4!} + \frac{e^{-5} 5^5}{5!} + \frac{e^{-5} 5^6}{6!} + \frac{e^{-5} 5^7}{7!} + \frac{e^{-5} 5^8}{8!}$$

$$P(4 \leq X \leq 8) = 0.17546737 + 0.17546737 + 0.146222808 + 0.104444863 +$$

$$+ 0.065278039$$

$$P(4 \leq X \leq 8) = 0.66688045$$

$$\lambda = 5 \times \frac{1}{2} = 2.5$$

$$P(X = x) = \frac{e^{-2.5} 2.5^x}{x!}, \quad x = 0, 1, 2, \dots$$

$$1 - P(X = 10) = \frac{e^{-2.5} 2.5^{10}}{10!}$$

$$P(X = 10) = 0.000215725$$

$$2 - P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$$

$$P(X < 3) = \frac{e^{-2.5} 2.5^0}{0!} + \frac{e^{-2.5} 2.5^1}{1!} + \frac{e^{-2.5} 2.5^2}{2!}$$

$$P(X < 3) = 0.082084999 + 0.205212497 + 0.256515621$$

$$P(X < 3) = 0.543813$$

$$3 - P(X > 1) = 1 - P(X \leq 1)$$

$$P(X > 1) = 1 - (P(X = 0) + P(X = 1))$$

$$P(X > 1) = 1 - (0.082084999 + 0.205212497)$$

$$P(X > 1) = 1 - 0.287297$$

$$P(X > 1) = 0.712703$$

$$4- P(4 \leq X \leq 8) = P(X = 4) + P(X = 5) + P(X = 6) + P(X = 7) + P(X = 8)$$

$$P(4 \leq X \leq 8) = \frac{e^{-2.5} 2.5^4}{4!} + \frac{e^{-2.5} 2.5^5}{5!} + \frac{e^{-2.5} 2.5^6}{6!} + \frac{e^{-2.5} 2.5^7}{7!} + \frac{e^{-2.5} 2.5^8}{8!}$$

$$P(4 \leq X \leq 8) = 0.133601886 + 0.066800943 + 0.027833726 + 0.009940617 + 0.003106443$$

$$P(4 \leq X \leq 8) = 0.241284$$

:

$$\lambda = 5 \times 2 = 10$$

$$P(X = x) = \frac{e^{-10} 10^x}{x!}, \quad x = 0, 1, 2, \dots$$

$$1- P(X = 10) = \frac{e^{-10} 10^{10}}{10!}$$

$$P(X = 10) = 0.125110036$$

$$2- P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$$

$$P(X < 3) = \frac{e^{-10} 10^0}{0!} + \frac{e^{-10} 10^1}{1!} + \frac{e^{-10} 10^2}{2!}$$

$$P(X < 3) = 0.0000454 + 0.000454 + 0.00227$$

$$P(X < 3) = 0.002769$$

$$3- P(X > 1) = 1 - P(X \leq 1)$$

$$P(X > 1) = 1 - (P(X = 0) + P(X = 1))$$

$$P(X > 1) = 1 - (0.0000454 + 0.000454)$$

$$P(X > 1) = 1 - 0.000499$$

$$P(X > 1) = 0.999501$$

$$4- P(4 \leq X \leq 8) = P(X = 4) + P(X = 5) + P(X = 6) + P(X = 7) + P(X = 8)$$

$$P(4 \leq X \leq 8) = \frac{e^{-10} 10^4}{4!} + \frac{e^{-10} 10^5}{5!} + \frac{e^{-10} 10^6}{6!} + \frac{e^{-10} 10^7}{7!} + \frac{e^{-10} 10^8}{8!}$$

$$P(4 \leq X \leq 8) = 0.01891664 + 0.03783327 + 0.06305546 + 0.09007923 + 0.11259903$$

$$P(4 \leq X \leq 8) = 0.322484$$

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:

$$\lambda = 2,$$

$$P(X = x) = \frac{e^{-2} 2^x}{x!}, \quad x = 0, 1, 2, \dots$$

$$P(X = 3) = \frac{e^{-2} 2^3}{3!}$$

$$P(X = 3) = 0.18044704$$

...

X

$$P(X = x) = \frac{\binom{a}{x} \binom{b}{n-x}}{\binom{N}{n}}, \quad x = 0, 1, 2, \dots, n$$

N -

n -

a + b = N -

:()

:

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-

-

a

X

:

b

:

$$P(X = x) = \frac{\binom{8}{x} \binom{40}{5-x}}{\binom{48}{5}}, \quad x = 0, 1, 2, 3, 4, 5$$

$$P(X = 0) = \frac{\binom{8}{0} \binom{40}{5-0}}{\binom{48}{5}}$$

$$P(X = 0) = \frac{1 \times 658008}{1712304}$$

$$P(X = 0) = 0.38$$

$$P(X = 1) = \frac{\binom{8}{1} \binom{40}{5-1}}{\binom{48}{5}}$$

$$P(X = 1) = \frac{8 \times 91390}{1712304}$$

$$P(X = 1) = 0.43$$

$$P(X \geq 2) = 1 - (P(X = 0) + P(X = 1))$$

$$P(X \geq 2) = 1 - (0.38 + 0.43)$$

$$P(X \geq 2) = 0.19$$

:()

a

X

:

b

:

$$P(X = x) = \frac{\binom{4}{x} \binom{76}{3-x}}{\binom{80}{3}}, \quad x = 0,1,2,3$$

:

$$P(X = 1) = \frac{\binom{4}{1} \binom{76}{3-1}}{\binom{80}{3}}$$

$$P(X = 1) = \frac{4 \times 2860}{82160}$$

$$P(X = 1) = 0.14$$

$$\mu$$

$$X$$

$$(X \approx N(\mu, \sigma^2))$$

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{x-\mu}{\sigma}\right)^2}$$

:

$$-\infty < X < \infty, \quad -\infty < \mu < \infty, \quad \sigma > 0$$

$$Z \approx N(0,1) \quad Z$$

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}, \quad -\infty < z < \infty$$

$$(\quad)$$

$$Z = \frac{x - \mu}{\sigma}$$

$$X \approx N(16,16)$$

$$Z = \frac{x - 16}{4}$$

$$Z$$

$$:(\quad)$$

$$1- P(z \leq 1.72), \quad 2- P(z \leq 1.07)$$

$$3- P(z \geq 0.29) \quad 4- P(-1.91 \leq z \leq 0.45)$$

$$1- P(z \leq 1.72) = 0.9573$$

$$2- P(z \leq 1.07) = 0.8577$$

$$3- P(z \geq 0.29) = 1- P(z < 0.29)$$

$$P(z \geq 0.29) = 1- 0.6141$$

$$P(z \geq 0.29) = 0.3859$$

$$4- P(-1.91 \leq z \leq 0.45) = P(z \leq 0.45) - P(z \leq -1.91)$$

$$P(-1.91 \leq z \leq 0.45) = 0.6736 - 0.0281$$

$$P(-1.91 \leq z \leq 0.45) = 0.6455$$

$$X \approx N(16,16) \quad :(\quad)$$

$$1- P(X \leq 14)$$

$$2- P(X \geq 22)$$

$$X$$

$$1- x = 14 \Rightarrow Z = \frac{x - \mu}{\sigma}$$

$$\Rightarrow z = \frac{14 - 16}{4} = -0.5$$

$$\Rightarrow P(X \leq 14) = P(Z \leq -0.5) = 0.3085$$

$$2- x = 22 \Rightarrow Z = \frac{x - \mu}{\sigma}$$

$$\Rightarrow z = \frac{22-16}{4} = 1.5$$

$$\Rightarrow P(X \geq 22) = P(Z \geq 1.5) = 1 - P(Z < 1.5)$$

$$\Rightarrow P(Z \geq 1.5) = 1 - 0.9332 = 0.0668$$

: ()

$$X \approx N(105, 100)$$

$$P(100 \leq X \leq 114) = P\left(\frac{100-105}{10} \leq \frac{X-\mu}{\sigma} \leq \frac{114-105}{10}\right)$$

$$P(100 \leq X \leq 114) = P(-0.5 \leq Z \leq 0.9)$$

$$P(100 \leq X \leq 114) = P(z \leq 0.9) - P(z \leq -0.5)$$

$$P(100 \leq X \leq 114) = 0.8159 - 0.3085$$

$$P(100 \leq X \leq 114) = 0.5074$$

$$f(t) = c \left(1 + \frac{t^2}{\nu}\right)^{-\nu + \frac{1}{2}}, \quad -\infty < t < \infty$$

$$\nu \qquad \qquad \qquad c \qquad \qquad \qquad \nu \qquad \qquad t$$

t

t

t

t

: ()

$$t(0.975, 20)$$

$$t(0.95, 5)$$

$$t(0.995, 12)$$

$$t(0.90, 7)$$

: t : :

$$t(0.975, 20) = 2.086$$

$$t(0.995, 12) = 3.055$$

$$t(0.95, 5) = 2.015$$

$$t(0.90, 7) = 1.415$$

:

:()

$$t(0.975, \nu) = 2.228$$

$$t(0.995, \nu) = 2.921$$

$$t(0.95, \nu) = 1.721$$

$$t(0.90, \nu) = 1.337$$

:

t

:

$$t(0.975, \nu) = 2.228 \Rightarrow \nu = 10$$

$$t(0.995, \nu) = 2.921 \Rightarrow \nu = 16$$

$$t(0.95, \nu) = 1.721 \Rightarrow \nu = 21$$

$$t(0.90, \nu) = 1.337 \Rightarrow \nu = 16$$

.F

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$$f(F) = \frac{cF^{\frac{(\nu_1-2)/2}{2}}}{(\nu_2 + \nu_1 F)^{\frac{(\nu_1+\nu_2)/2}{2}}}, \quad F > 0$$

 ν_2 ν_1 $F(\nu_1, \nu_2)$

F

c

F

(

 $\alpha) F(\alpha, \nu_1, \nu_2)$

:()

$$F(0.01, 11, 15)$$

$$F(0.05, 10, 7)$$

$$F(0.01, 11, 15) = 0.235$$

$$F(0.05, 10, 7) = 0.318$$

%

Z

- $P(Z < 1.8)$
- $P(Z > -0.5)$
- $P(-0.2 < Z < 0.5)$

X

%

X

X

: t - $t (0.95, 20)$ $t (0.90, 28)$ $t (0.99, 12)$ $t (0.975, 7)$ $t (0.995, 13)$: b -
 $(b, 5) = 2.015$ t $t (b, 20) = 1.325$ $t (b, 23) = 2.069$ $t (b, 12) = 2.681$ $t (b, 15) = 2.131$

: -

 $F (0.01, 7, 12)$ $F (0.05, 12, 5)$ $F (0.01, 5, 8)$ $F (0.05, 5, 5)$: b - $F (b, 8, 9) = 3.23$ $F (b, 9, 11) = 4.63$ $F (b, 3, 24) = 3.72$ $F (b, 2, 24) = 3.4$

(-)

. Statistical Hypothesis

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Parametric Statistical Test of Hypothesis

H_o	Null Hypothesis	:
H_1	Alternative Hypothesis	:

Significant "

β

α

H_o

)

α

) α

.(... ,

H_o

β

:

H_o	H_o	
α		H_o
	β	H_o

(

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H_o

: ()

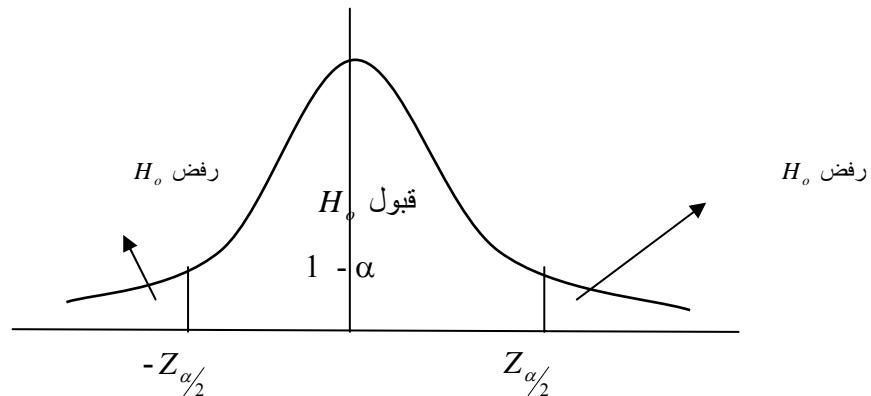
(1- α)

: α

: () μ

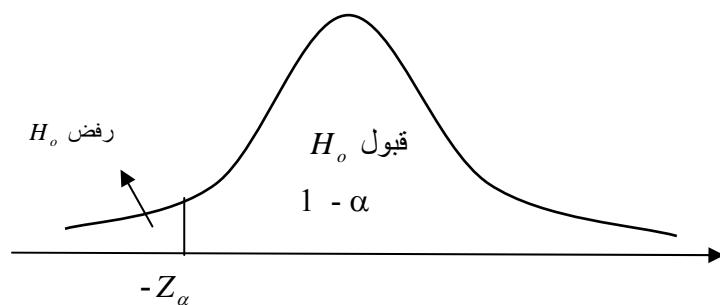
$$H_0: \mu = \mu_o$$

$$H_1: \mu \neq \mu_o$$



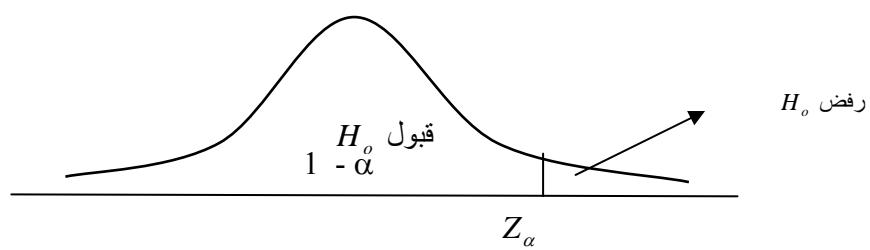
$$H_0: \mu = \mu_o$$

$$H_1: \mu < \mu_o$$



$$H_0: \mu = \mu_o$$

$$H_1: \mu > \mu_o$$



μ

$$Z \sim N(0,1)$$

̄x

Z

μ_o

n

() α

Z_α - Z_{α/2} Z_{α/2}

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$$H_o: \mu = 160$$

$$H_1: \mu \neq 160$$

$$Z = \frac{\bar{x} - \mu_o}{\sigma / \sqrt{n}}$$

$$\bar{x} = 155, \quad \mu_o = 160, \quad \sigma = 5, \quad n = 64$$

$$\alpha = 0.05$$

$$\Rightarrow Z = \frac{155 - 160}{\frac{5}{\sqrt{64}}},$$

$$Z = -8 \quad (1)$$

Z

Z (α = 0.05) -

$Z_{\alpha/2} = Z_{0.05/2} = Z_{0.025}$
 $Z_{0.025} = 1.96$

$-Z_{\alpha/2} = -1.96$

Z Z Z

$\cdot \mu = 160$

Z (α = 0.01) -

$Z_{\alpha/2} = Z_{0.01/2} = Z_{0.005}$
 $Z_{0.005} = 2.58$

$-Z_{\alpha/2} = -2.58$

Z Z Z

$\cdot \mu = 160$

:()

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$H_o: \mu = 240$
 $H_1: \mu < 240$

Z -

$Z = \frac{\bar{x} - \mu_o}{\sigma/\sqrt{n}}$

$$\begin{array}{llll}
 \bar{x} = 235, & \mu_o = 240, & \sigma = 18, & n = 9 \\
 \alpha = 0.01 & & & \\
 \Rightarrow Z = \frac{235 - 240}{\frac{18}{\sqrt{9}}}, & & & \\
 Z = -0.83 & (1) & & \\
 \end{array}$$

$$\begin{array}{llll}
 -Z_\alpha = -Z_{0.1} & & & \\
 -Z_{0.1} = -1.28 & (2) & & \\
 Z & Z & Z & (\) \quad (\) \\
 \end{array}$$

$$\cdot \mu = 240$$

n

v

$$v = n - k$$

k

$$\begin{array}{llll}
 \vdots & t & Z & (\quad > \quad) \\
 t = \frac{\bar{x} - \mu_o}{\sqrt{\frac{S}{n}}} & t & & \\
 t & t & S & \\
 \alpha/2 & \alpha & & (v = n - 1) \\
 \vdots & & & \\
 \end{array}$$

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%

: - :

 $H_o: \mu = 15$ $H_1: \mu > 15$

t -

$$t = \frac{\bar{x} - \mu_o}{\sqrt{S/n}}$$

:

$$\bar{x} = 17, \quad \mu_o = 15, \quad S = 2,$$

$$n = 7$$

$$\alpha = 0.05$$

$$\Rightarrow t = \frac{17 - 15}{\sqrt{2/7}},$$

$$t = 2.65 \quad (1)$$

t

. -

$$t \quad (\alpha = 0.05)$$

:

$$t_{(n-1, \alpha)} = t_{(7-1, 0.05)}$$

$$t_{(6, 0.05)} = 1.943 \quad (2)$$

t

t

t

() ()

.($\alpha = 0.05$)

$$\begin{aligned}
 H_o: \mu_1 &= \mu_2 \\
 H_1: \mu_1 &< \mu_2
 \end{aligned}
 \quad Z$$

$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_o}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

$$Z = \frac{(69 - 74) - 0}{\sqrt{\frac{230}{60} + \frac{215}{85}}}$$

$$Z = \frac{-5}{\sqrt{6.36}}$$

$$Z = -1.98, \quad (1)$$

Z

(α = 0.05)

$$-Z_\alpha = -Z_{0.05}$$

$$-Z_{0.05} = -1.65 \quad (2)$$

Z

Z

Z

() ()

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_o}{\sqrt{S_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

:

. (ν = n₁ + n₂ - 2)

t

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$$H_o: \mu_1 = \mu_2$$

$$H_1: \mu_1 \neq \mu_2$$

$$: \quad S_p^2 \quad -$$

$$1- S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}$$

$$S_p^2 = \frac{(11-1)(7)^2 + (6-1)(5)^2}{11+6-2}$$

$$S_p^2 = \frac{10 \times 49 + 5 \times 25}{15}$$

$$S_p^2 = 41 \quad (1)$$

$$. \quad t \quad -$$

$$2- t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_o}{\sqrt{S_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

$$t = \frac{(80-75)-0}{\sqrt{41(\frac{1}{11} + \frac{1}{6})}}$$

$$t = \frac{5}{\sqrt{10.56}}$$

$$t = 1.54 \quad (2)$$

$$. \quad t \quad -$$

$$t \quad (\alpha = 0.05)$$

:

$$t_{(n_1+n_2-2, \alpha/2)} = t_{(11+6-2, 0.025)}$$

$$t_{(15, 0.025)} = \pm 2.131 \quad (3)$$

$$t \quad t \quad t \quad () \quad ()$$

%

$$(v = n - 1)$$

$$t = \frac{\bar{d} - \bar{D}_o}{s_d / \sqrt{n}}$$

d

S_d

.()

11

$$\overline{D}_o$$

$$H_o : \bar{D}_o = 0, \quad \Leftrightarrow \quad H_o : \bar{D}_o = \mu_1 - \mu_2$$

$$H_1: \overline{D}_o \neq 0,$$

$$\text{or} \quad H_1: \bar{D}_o < 0,$$

or $H_1: \bar{D}_o > 0$

t

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$$H_o : \overline{D}_o = 0$$

$$H_1: \overline{D}_o \neq 0$$

$$t = \frac{\bar{d} - \bar{D}_o}{s_d / \sqrt{n}}$$

$$1 - \bar{D}_o = 0$$

$$2 - n = 7$$

(d^2)	(d)	
	-	
	-	
	-	
	-	
	-	
	-	
	-	
	-	

$$\bar{d} = \frac{\sum_{i=1}^n d_i}{n}$$

$$\bar{d} = \frac{-4343}{7}$$

$$\bar{d} = -620.43 \quad (1)$$

$$s_d = \frac{1}{n-1} \sqrt{\sum_{i=1}^n d_i^2 - \frac{(\sum_{i=1}^n d_i)^2}{n}}$$

$$s_d = \frac{1}{7-1} \sqrt{3595551 - \frac{(-4343)^2}{7}}$$

$$s_d = \frac{1}{6} \sqrt{901029.71}$$

$$s_d = \frac{1}{6} (949.23)$$

$$s_d = 158.2 \quad (2)$$

$$t = \frac{-620.43}{158.2 / \sqrt{7}}$$

$$t = -10.38 \quad (3)$$

t ($\alpha = 0.05$)

:

$t_{(n-1, \alpha/2)} = t_{(6, 0.025)}$
 $t_{(6, 0.025)} = \pm 2.448$ (4)

t ($)$ ($)$ $\%$

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 π

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p π

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$H_o: \pi = \pi_o$,
 $H_1: \pi \neq \pi_o$,
 or $H_1: \pi < \pi_o$,
 or $H_1: \pi > \pi_o$

$$n \frac{p - \pi_o}{\sqrt{\frac{\pi_o(1 - \pi_o)}{n}}}$$

Z

$$Z = \frac{p - \pi_o}{\sqrt{\frac{\pi_o(1 - \pi_o)}{n}}}$$

Z

:

$:()$
 $\%$
 $\%$

:

$H_o: \pi = 0.06$ $H_1: \pi > 0.06$

: : - -

$$Z = \frac{p - \pi_o}{\sqrt{\frac{\pi_o(1 - \pi_o)}{n}}}$$

$$p = \frac{20}{180}$$

$$p = 0.11$$

$$Z = \frac{0.11 - 0.06}{\sqrt{\frac{0.06(1 - 0.06)}{180}}}$$

$$Z = \frac{0.05}{\sqrt{\frac{0.06 \times 0.94}{180}}}$$

$$Z = \frac{0.05}{0.02}$$

$$Z = 2.5 \quad (1)$$

: % Z

$$Z_\alpha = Z_{0.01}$$

$$Z_{0.01} = 2.33 \quad (2)$$

% Z () ()

$$(\pi_1 - \pi_2)$$

 $p_2 \quad p_1 \quad \dots \quad \dots$ $n_2 \quad n_1$: Z $p_1 - p_2$

$$Z = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)_0}{\sqrt{\bar{p}(1 - \bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \quad (1)$$

$$\bar{P} = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} \quad ()$$

$$H_o: (\pi_1 - \pi_2) = 0$$

$$H_1: (\pi_1 - \pi_2) \neq 0,$$

$$\text{or, } H_1: (\pi_1 - \pi_2) > 0$$

$$\text{or, } H_1: (\pi_1 - \pi_2) < 0$$

$$H_o: (\pi_1 - \pi_2) = \pi, \quad \pi \neq 0$$

$$Z = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)_0}{\sqrt{\frac{p_1(1 - p_1)}{n_1} + \frac{p_2(1 - p_2)}{n_2}}}, \quad (2)$$

$$H_1: (\pi_1 - \pi_2) \neq \pi,$$

$$\text{or, } H_1: (\pi_1 - \pi_2) > \pi$$

$$\text{or, } H_1: (\pi_1 - \pi_2) < \pi$$

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$$H_o: (\pi_1 - \pi_2) = 0,$$

$$H_1: (\pi_1 - \pi_2) > 0$$

: Z -

$$Z = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)_0}{\sqrt{\bar{p}(1 - \bar{p})(\frac{1}{n_1} + \frac{1}{n_2})}}$$

$$p_1 = \frac{80}{100} = 0.8$$

$$p_2 = \frac{62}{100} = 0.62$$

$$\bar{P} = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

$$\bar{P} = \frac{100 \times 0.8 + 100 \times 0.62}{100 + 100}$$

$$\bar{P} = \frac{142}{200}$$

$$\bar{P} = 0.71$$

$$Z = \frac{0.8 - 0.62 - 0}{\sqrt{0.71 \times 0.29 (\frac{1}{100} + \frac{1}{100})}}$$

$$Z = \frac{0.18}{\sqrt{0.21 \times 0.02}}$$

$$Z = 2.78 \quad (1)$$

. Z -

$$Z_\alpha = Z_{0.05}$$

$$Z_{0.05} = 1.65 \quad (2)$$

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$$H_o: (\pi_1 - \pi_2) = 0.04,$$

$$H_1: (\pi_1 - \pi_2) < 0.04$$

: Z -

$$Z = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)_0}{\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}}$$

$$p_1 = \frac{12}{100} = 0.12$$

$$p_2 = \frac{15}{150} = 0.1$$

$$Z = \frac{(0.12 - 0.1) - 0.04}{\sqrt{\frac{0.12 \times 0.88}{100} + \frac{0.1 \times 0.09}{150}}}$$

$$Z = \frac{-0.02}{\sqrt{0.0011 + 0.0006}}$$

$$Z = -0.49 \quad (1)$$

. Z -

$$Z_\alpha = -Z_{0.05}$$

$$Z_{0.05} = -1.65 \quad (2)$$

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	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

dfp	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	4.3178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
21	0.256580	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.319460	1.713872	2.06866	2.49987	2.80734	3.7676
24	0.256173	0.684850	1.317836	1.710882	2.06390	2.49216	2.79694	3.7454
25	0.256060	0.684430	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896
28	0.255768	0.683353	1.312527	1.701131	2.04841	2.46714	2.76326	3.6739
29	0.255684	0.683044	1.311434	1.699127	2.04523	2.46202	2.75639	3.6594
30	0.255605	0.682756	1.310415	1.697261	2.04227	2.45726	2.75000	3.6460
inf	0.253347	0.674490	1.281552	1.644854	1.95996	2.32635	2.57583	3.2905

Chi-Square Table

df/ area	.995	.990	.975	.950	.900	.750	.500
1	0.00004	0.00016	0.00098	0.00393	0.01579	0.10153	0.45494
2	0.01003	0.02010	0.05064	0.10259	0.21072	0.57536	1.38629
3	0.07172	0.11483	0.21580	0.35185	0.58437	1.21253	2.36597
4	0.20699	0.29711	0.48442	0.71072	1.06362	1.92256	3.35669
5	0.41174	0.55430	0.83121	1.14548	1.61031	2.67460	4.35146
6	0.67573	0.87209	1.23734	1.63538	2.20413	3.45460	5.34812
7	0.98926	1.23904	1.68987	2.16735	2.83311	4.25485	6.34581
8	1.34441	1.64650	2.17973	2.73264	3.48954	5.07064	7.34412
9	1.73493	2.08790	2.70039	3.32511	4.16816	5.89883	8.34283
10	2.15586	2.55821	3.24697	3.94030	4.86518	6.73720	9.34182
11	2.60322	3.05348	3.81575	4.57481	5.57778	7.58414	10.34100
12	3.07382	3.57057	4.40379	5.22603	6.30380	8.43842	11.34032
13	3.56503	4.10692	5.00875	5.89186	7.04150	9.29907	12.33976
14	4.07467	4.66043	5.62873	6.57063	7.78953	10.16531	13.33927
15	4.60092	5.22935	6.26214	7.26094	8.54676	11.03654	14.33886
16	5.14221	5.81221	6.90766	7.96165	9.31224	11.91222	15.33850
17	5.69722	6.40776	7.56419	8.67176	10.08519	12.79193	16.33818
18	6.26480	7.01491	8.23075	9.39046	10.86494	13.67529	17.33790
19	6.84397	7.63273	8.90652	10.11701	11.65091	14.56200	18.33765
20	7.43384	8.26040	9.59078	10.85081	12.44261	15.45177	19.33743
21	8.03365	8.89720	10.28290	11.59131	13.23960	16.34438	20.33723
22	8.64272	9.54249	10.98232	12.33801	14.04149	17.23962	21.33704
23	9.26042	10.19572	11.68855	13.09051	14.84796	18.13730	22.33688
24	9.88623	10.85636	12.40115	13.84843	15.65868	19.03725	23.33673
25	10.51965	11.52398	13.11972	14.61141	16.47341	19.93934	24.33659
26	11.16024	12.19815	13.84390	15.37916	17.29188	20.84343	25.33646
27	11.80759	12.87850	14.57338	16.15140	18.11390	21.74940	26.33634
28	12.46134	13.56471	15.30786	16.92788	18.93924	22.65716	27.33623
29	13.12115	14.25645	16.04707	17.70837	19.76774	23.56659	28.33613
30	13.78672	14.95346	16.79077	18.49266	20.59923	24.47761	29.33603

Chi-Square Table (Cont.)

df/ area	.250	.100	.050	.025	.010	.005
1	1.32330	2.70554	3.84146	5.02389	6.63490	7.87944
2	2.77259	4.60517	5.99146	7.37776	9.21034	10.59663
3	4.10834	6.25139	7.81473	9.34840	11.34487	12.83816
4	5.38527	7.77944	9.48773	11.14329	13.27670	14.86026
5	6.62568	9.23636	11.07050	12.83250	15.08627	16.74960
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6	7.84080	10.64464	12.59159	14.44938	16.81189	18.54758
7	9.03715	12.01704	14.06714	16.01276	18.47531	20.27774
8	10.21885	13.36157	15.50731	17.53455	20.09024	21.95495
9	11.38875	14.68366	16.91898	19.02277	21.66599	23.58935
10	12.54886	15.98718	18.30704	20.48318	23.20925	25.18818
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11	13.70069	17.27501	19.67514	21.92005	24.72497	26.75685
12	14.84540	18.54935	21.02607	23.33666	26.21697	28.29952
13	15.98391	19.81193	22.36203	24.73560	27.68825	29.81947
14	17.11693	21.06414	23.68479	26.11895	29.14124	31.31935
15	18.24509	22.30713	24.99579	27.48839	30.57791	32.80132
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16	19.36886	23.54183	26.29623	28.84535	31.99993	34.26719
17	20.48868	24.76904	27.58711	30.19101	33.40866	35.71847
18	21.60489	25.98942	28.86930	31.52638	34.80531	37.15645
19	22.71781	27.20357	30.14353	32.85233	36.19087	38.58226
20	23.82769	28.41198	31.41043	34.16961	37.56623	39.99685
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21	24.93478	29.61509	32.67057	35.47888	38.93217	41.40106
22	26.03927	30.81328	33.92444	36.78071	40.28936	42.79565
23	27.14134	32.00690	35.17246	38.07563	41.63840	44.18128
24	28.24115	33.19624	36.41503	39.36408	42.97982	45.55851
25	29.33885	34.38159	37.65248	40.64647	44.31410	46.92789
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26	30.43457	35.56317	38.88514	41.92317	45.64168	48.28988
27	31.52841	36.74122	40.11327	43.19451	46.96294	49.64492
28	32.62049	37.91592	41.33714	44.46079	48.27824	50.99338
29	33.71091	39.08747	42.55697	45.72229	49.58788	52.33562
30	34.79974	40.25602	43.77297	46.97924	50.89218	53.67196

df2/ df1	1	2	3	4	5	6	7	8	9	10
1	39.86346	49.50000	53.59324	55.83296	57.24008	58.20442	58.90595	59.43898	59.85759	60.19498
2	8.52632	9.00000	9.16179	9.24342	9.29263	9.32553	9.34908	9.36677	9.38054	9.39157
3	5.53832	5.46238	5.39077	5.34264	5.30916	5.28473	5.26619	5.25167	5.24000	5.23041
4	4.54477	4.32456	4.19086	4.10725	4.05058	4.00975	3.97897	3.95494	3.93567	3.91988
5	4.06042	3.77972	3.61948	3.52020	3.45298	3.40451	3.36790	3.33928	3.31628	3.29740
6	3.77595	3.46330	3.28876	3.18076	3.10751	3.05455	3.01446	2.98304	2.95774	2.93693
7	3.58943	3.25744	3.07407	2.96053	2.88334	2.82739	2.78493	2.75158	2.72468	2.70251
8	3.45792	3.11312	2.92380	2.80643	2.72645	2.66833	2.62413	2.58935	2.56124	2.53804
9	3.36030	3.00645	2.81286	2.69268	2.61061	2.55086	2.50531	2.46941	2.44034	2.41632
10	3.28502	2.92447	2.72767	2.60534	2.52164	2.46058	2.41397	2.37715	2.34731	2.32260
11	3.22520	2.85951	2.66023	2.53619	2.45118	2.38907	2.34157	2.30400	2.27350	2.24823
12	3.17655	2.80680	2.60552	2.48010	2.39402	2.33102	2.28278	2.24457	2.21352	2.18776
13	3.13621	2.76317	2.56027	2.43371	2.34672	2.28298	2.23410	2.19535	2.16382	2.13763
14	3.10221	2.72647	2.52222	2.39469	2.30694	2.24256	2.19313	2.15390	2.12195	2.09540
15	3.07319	2.69517	2.48979	2.36143	2.27302	2.20808	2.15818	2.11853	2.08621	2.05932
16	3.04811	2.66817	2.46181	2.33274	2.24376	2.17833	2.12800	2.08798	2.05533	2.02815
17	3.02623	2.64464	2.43743	2.30775	2.21825	2.15239	2.10169	2.06134	2.02839	2.00094
18	3.00698	2.62395	2.41601	2.28577	2.19583	2.12958	2.07854	2.03789	2.00467	1.97698
19	2.98990	2.60561	2.39702	2.26630	2.17596	2.10936	2.05802	2.01710	1.98364	1.95573
20	2.97465	2.58925	2.38009	2.24893	2.15823	2.09132	2.03970	1.99853	1.96485	1.93674
21	2.96096	2.57457	2.36489	2.23334	2.14231	2.07512	2.02325	1.98186	1.94797	1.91967
22	2.94858	2.56131	2.35117	2.21927	2.12794	2.06050	2.00840	1.96680	1.93273	1.90425
23	2.93736	2.54929	2.33873	2.20651	2.11491	2.04723	1.99492	1.95312	1.91888	1.89025
24	2.92712	2.53833	2.32739	2.19488	2.10303	2.03513	1.98263	1.94066	1.90625	1.87748
25	2.91774	2.52831	2.31702	2.18424	2.09216	2.02406	1.97138	1.92925	1.89469	1.86578
26	2.90913	2.51910	2.30749	2.17447	2.08218	2.01389	1.96104	1.91876	1.88407	1.85503
27	2.90119	2.51061	2.29871	2.16546	2.07298	2.00452	1.95151	1.90909	1.87427	1.84511
28	2.89385	2.50276	2.29060	2.15714	2.06447	1.99585	1.94270	1.90014	1.86520	1.83593
29	2.88703	2.49548	2.28307	2.14941	2.05658	1.98781	1.93452	1.89184	1.85679	1.82741
30	2.88069	2.48872	2.27607	2.14223	2.04925	1.98033	1.92692	1.88412	1.84896	1.81949
40	2.83535	2.44037	2.22609	2.09095	1.99682	1.92688	1.87252	1.82886	1.79290	1.76269
60	2.79107	2.39325	2.17741	2.04099	1.94571	1.87472	1.81939	1.77483	1.73802	1.70701
120	2.74781	2.34734	2.12999	1.99230	1.89587	1.82381	1.76748	1.72196	1.68425	1.65238
inf	2.70554	2.30259	2.08380	1.94486	1.84727	1.77411	1.71672	1.67020	1.63152	1.59872

df2/ df1	12	15	20	24	30	40	60	120	INF
1	60.70521	61.22034	61.74029	62.00205	62.26497	62.52905	62.79428	63.06064	63.32812
2	9.40813	9.42471	9.44131	9.44962	9.45793	9.46624	9.47456	9.48289	9.49122
3	5.21562	5.20031	5.18448	5.17636	5.16811	5.15972	5.15119	5.14251	5.13370
4	3.89553	3.87036	3.84434	3.83099	3.81742	3.80361	3.78957	3.77527	3.76073
5	3.26824	3.23801	3.20665	3.19052	3.17408	3.15732	3.14023	3.12279	3.10500
6	2.90472	2.87122	2.83634	2.81834	2.79996	2.78117	2.76195	2.74229	2.72216
7	2.66811	2.63223	2.59473	2.57533	2.55546	2.53510	2.51422	2.49279	2.47079
8	2.50196	2.46422	2.42464	2.40410	2.38302	2.36136	2.33910	2.31618	2.29257
9	2.37888	2.33962	2.29832	2.27683	2.25472	2.23196	2.20849	2.18427	2.15923
10	2.28405	2.24351	2.20074	2.17843	2.15543	2.13169	2.10716	2.08176	2.05542
11	2.20873	2.16709	2.12305	2.10001	2.07621	2.05161	2.02612	1.99965	1.97211
12	2.14744	2.10485	2.05968	2.03599	2.01149	1.98610	1.95973	1.93228	1.90361
13	2.09659	2.05316	2.00698	1.98272	1.95757	1.93147	1.90429	1.87591	1.84620
14	2.05371	2.00953	1.96245	1.93766	1.91193	1.88516	1.85723	1.82800	1.79728
15	2.01707	1.97222	1.92431	1.89904	1.87277	1.84539	1.81676	1.78672	1.75505
16	1.98539	1.93992	1.89127	1.86556	1.83879	1.81084	1.78156	1.75075	1.71817
17	1.95772	1.91169	1.86236	1.83624	1.80901	1.78053	1.75063	1.71909	1.68564
18	1.93334	1.88681	1.83685	1.81035	1.78269	1.75371	1.72322	1.69099	1.65671
19	1.91170	1.86471	1.81416	1.78731	1.75924	1.72979	1.69876	1.66587	1.63077
20	1.89236	1.84494	1.79384	1.76667	1.73822	1.70833	1.67678	1.64326	1.60738
21	1.87497	1.82715	1.77555	1.74807	1.71927	1.68896	1.65691	1.62278	1.58615
22	1.85925	1.81106	1.75899	1.73122	1.70208	1.67138	1.63885	1.60415	1.56678
23	1.84497	1.79643	1.74392	1.71588	1.68643	1.65535	1.62237	1.58711	1.54903
24	1.83194	1.78308	1.73015	1.70185	1.67210	1.64067	1.60726	1.57146	1.53270
25	1.82000	1.77083	1.71752	1.68898	1.65895	1.62718	1.59335	1.55703	1.51760
26	1.80902	1.75957	1.70589	1.67712	1.64682	1.61472	1.58050	1.54368	1.50360
27	1.79889	1.74917	1.69514	1.66616	1.63560	1.60320	1.56859	1.53129	1.49057
28	1.78951	1.73954	1.68519	1.65600	1.62519	1.59250	1.55753	1.51976	1.47841
29	1.78081	1.73060	1.67593	1.64655	1.61551	1.58253	1.54721	1.50899	1.46704
30	1.77270	1.72227	1.66731	1.63774	1.60648	1.57323	1.53757	1.49891	1.45636
40	1.71456	1.66241	1.60515	1.57411	1.54108	1.50562	1.46716	1.42476	1.37691
60	1.65743	1.60337	1.54349	1.51072	1.47554	1.43734	1.39520	1.34757	1.29146
120	1.60120	1.54500	1.48207	1.44723	1.40938	1.36760	1.32034	1.26457	1.19256
inf	1.54578	1.48714	1.42060	1.38318	1.34187	1.29513	1.23995	1.16860	1.00000

F Table for alpha=.05

df2/ df1	1	2	3	4	5	6	7	8	9	10
1	161.4476	199.5000	215.7073	224.5832	230.1619	233.9860	236.7684	238.8827	240.5433	241.8817
2	18.5128	19.0000	19.1643	19.2468	19.2964	19.3295	19.3532	19.3710	19.3848	19.3959
3	10.1280	9.5521	9.2766	9.1172	9.0135	8.9406	8.8867	8.8452	8.8123	8.7855
4	7.7086	6.9443	6.5914	6.3882	6.2561	6.1631	6.0942	6.0410	5.9988	5.9644
5	6.6079	5.7861	5.4095	5.1922	5.0503	4.9503	4.8759	4.8183	4.7725	4.7351
6	5.9874	5.1433	4.7571	4.5337	4.3874	4.2839	4.2067	4.1468	4.0990	4.0600
7	5.5914	4.7374	4.3468	4.1203	3.9715	3.8660	3.7870	3.7257	3.6767	3.6365
8	5.3177	4.4590	4.0662	3.8379	3.6875	3.5806	3.5005	3.4381	3.3881	3.3472
9	5.1174	4.2565	3.8625	3.6331	3.4817	3.3738	3.2927	3.2296	3.1789	3.1373
10	4.9646	4.1028	3.7083	3.4780	3.3258	3.2172	3.1355	3.0717	3.0204	2.9782
11	4.8443	3.9823	3.5874	3.3567	3.2039	3.0946	3.0123	2.9480	2.8962	2.8536
12	4.7472	3.8853	3.4903	3.2592	3.1059	2.9961	2.9134	2.8486	2.7964	2.7534
13	4.6672	3.8056	3.4105	3.1791	3.0254	2.9153	2.8321	2.7669	2.7144	2.6710
14	4.6001	3.7389	3.3439	3.1122	2.9582	2.8477	2.7642	2.6987	2.6458	2.6022
15	4.5431	3.6823	3.2874	3.0556	2.9013	2.7905	2.7066	2.6408	2.5876	2.5437
16	4.4940	3.6337	3.2389	3.0069	2.8524	2.7413	2.6572	2.5911	2.5377	2.4935
17	4.4513	3.5915	3.1968	2.9647	2.8100	2.6987	2.6143	2.5480	2.4943	2.4499
18	4.4139	3.5546	3.1599	2.9277	2.7729	2.6613	2.5767	2.5102	2.4563	2.4117
19	4.3807	3.5219	3.1274	2.8951	2.7401	2.6283	2.5435	2.4768	2.4227	2.3779
20	4.3512	3.4928	3.0984	2.8661	2.7109	2.5990	2.5140	2.4471	2.3928	2.3479
21	4.3248	3.4668	3.0725	2.8401	2.6848	2.5727	2.4876	2.4205	2.3660	2.3210
22	4.3009	3.4434	3.0491	2.8167	2.6613	2.5491	2.4638	2.3965	2.3419	2.2967
23	4.2793	3.4221	3.0280	2.7955	2.6400	2.5277	2.4422	2.3748	2.3201	2.2747
24	4.2597	3.4028	3.0088	2.7763	2.6207	2.5082	2.4226	2.3551	2.3002	2.2547
25	4.2417	3.3852	2.9912	2.7587	2.6030	2.4904	2.4047	2.3371	2.2821	2.2365
26	4.2252	3.3690	2.9752	2.7426	2.5868	2.4741	2.3883	2.3205	2.2655	2.2197
27	4.2100	3.3541	2.9604	2.7278	2.5719	2.4591	2.3732	2.3053	2.2501	2.2043
28	4.1960	3.3404	2.9467	2.7141	2.5581	2.4453	2.3593	2.2913	2.2360	2.1900
29	4.1830	3.3277	2.9340	2.7014	2.5454	2.4324	2.3463	2.2783	2.2229	2.1768
30	4.1709	3.3158	2.9223	2.6896	2.5336	2.4205	2.3343	2.2662	2.2107	2.1646
40	4.0847	3.2317	2.8387	2.6060	2.4495	2.3359	2.2490	2.1802	2.1240	2.0772
60	4.0012	3.1504	2.7581	2.5252	2.3683	2.2541	2.1665	2.0970	2.0401	1.9926
120	3.9201	3.0718	2.6802	2.4472	2.2899	2.1750	2.0868	2.0164	1.9588	1.9105
inf	3.8415	2.9957	2.6049	2.3719	2.2141	2.0986	2.0096	1.9384	1.8799	1.8307

F Table for alpha=.05 (Cont.)

df2/ df1	12	15	20	24	30	40	60	120	INF
1	243.9060	245.9499	248.0131	249.0518	250.0951	251.1432	252.1957	253.2529	254.3144
2	19.4125	19.4291	19.4458	19.4541	19.4624	19.4707	19.4791	19.4874	19.4957
3	8.7446	8.7029	8.6602	8.6385	8.6166	8.5944	8.5720	8.5494	8.5264
4	5.9117	5.8578	5.8025	5.7744	5.7459	5.7170	5.6877	5.6581	5.6281
5	4.6777	4.6188	4.5581	4.5272	4.4957	4.4638	4.4314	4.3985	4.3650
6	3.9999	3.9381	3.8742	3.8415	3.8082	3.7743	3.7398	3.7047	3.6689
7	3.5747	3.5107	3.4445	3.4105	3.3758	3.3404	3.3043	3.2674	3.2298
8	3.2839	3.2184	3.1503	3.1152	3.0794	3.0428	3.0053	2.9669	2.9276
9	3.0729	3.0061	2.9365	2.9005	2.8637	2.8259	2.7872	2.7475	2.7067
10	2.9130	2.8450	2.7740	2.7372	2.6996	2.6609	2.6211	2.5801	2.5379
11	2.7876	2.7186	2.6464	2.6090	2.5705	2.5309	2.4901	2.4480	2.4045
12	2.6866	2.6169	2.5436	2.5055	2.4663	2.4259	2.3842	2.3410	2.2962
13	2.6037	2.5331	2.4589	2.4202	2.3803	2.3392	2.2966	2.2524	2.2064
14	2.5342	2.4630	2.3879	2.3487	2.3082	2.2664	2.2229	2.1778	2.1307
15	2.4753	2.4034	2.3275	2.2878	2.2468	2.2043	2.1601	2.1141	2.0658
16	2.4247	2.3522	2.2756	2.2354	2.1938	2.1507	2.1058	2.0589	2.0096
17	2.3807	2.3077	2.2304	2.1898	2.1477	2.1040	2.0584	2.0107	1.9604
18	2.3421	2.2686	2.1906	2.1497	2.1071	2.0629	2.0166	1.9681	1.9168
19	2.3080	2.2341	2.1555	2.1141	2.0712	2.0264	1.9795	1.9302	1.8780
20	2.2776	2.2033	2.1242	2.0825	2.0391	1.9938	1.9464	1.8963	1.8432
21	2.2504	2.1757	2.0960	2.0540	2.0102	1.9645	1.9165	1.8657	1.8117
22	2.2258	2.1508	2.0707	2.0283	1.9842	1.9380	1.8894	1.8380	1.7831
23	2.2036	2.1282	2.0476	2.0050	1.9605	1.9139	1.8648	1.8128	1.7570
24	2.1834	2.1077	2.0267	1.9838	1.9390	1.8920	1.8424	1.7896	1.7330
25	2.1649	2.0889	2.0075	1.9643	1.9192	1.8718	1.8217	1.7684	1.7110
26	2.1479	2.0716	1.9898	1.9464	1.9010	1.8533	1.8027	1.7488	1.6906
27	2.1323	2.0558	1.9736	1.9299	1.8842	1.8361	1.7851	1.7306	1.6717
28	2.1179	2.0411	1.9586	1.9147	1.8687	1.8203	1.7689	1.7138	1.6541
29	2.1045	2.0275	1.9446	1.9005	1.8543	1.8055	1.7537	1.6981	1.6376
30	2.0921	2.0148	1.9317	1.8874	1.8409	1.7918	1.7396	1.6835	1.6223
40	2.0035	1.9245	1.8389	1.7929	1.7444	1.6928	1.6373	1.5766	1.5089
60	1.9174	1.8364	1.7480	1.7001	1.6491	1.5943	1.5343	1.4673	1.3893
120	1.8337	1.7505	1.6587	1.6084	1.5543	1.4952	1.4290	1.3519	1.2539
inf	1.7522	1.6664	1.5705	1.5173	1.4591	1.3940	1.3180	1.2214	1.0000

F Table for alpha=.025

df2/ df1	1	2	3	4	5	6	7	8	9	10
1	647.7890	799.5000	864.1630	899.5833	921.8479	937.1111	948.2169	956.6562	963.2846	968.6274
2	38.5063	39.0000	39.1655	39.2484	39.2982	39.3315	39.3552	39.3730	39.3869	39.3980
3	17.4434	16.0441	15.4392	15.1010	14.8848	14.7347	14.6244	14.5399	14.4731	14.4189
4	12.2179	10.6491	9.9792	9.6045	9.3645	9.1973	9.0741	8.9796	8.9047	8.8439
5	10.0070	8.4336	7.7636	7.3879	7.1464	6.9777	6.8531	6.7572	6.6811	6.6192
6	8.8131	7.2599	6.5988	6.2272	5.9876	5.8198	5.6955	5.5996	5.5234	5.4613
7	8.0727	6.5415	5.8898	5.5226	5.2852	5.1186	4.9949	4.8993	4.8232	4.7611
8	7.5709	6.0595	5.4160	5.0526	4.8173	4.6517	4.5286	4.4333	4.3572	4.2951
9	7.2093	5.7147	5.0781	4.7181	4.4844	4.3197	4.1970	4.1020	4.0260	3.9639
10	6.9367	5.4564	4.8256	4.4683	4.2361	4.0721	3.9498	3.8549	3.7790	3.7168
11	6.7241	5.2559	4.6300	4.2751	4.0440	3.8807	3.7586	3.6638	3.5879	3.5257
12	6.5538	5.0959	4.4742	4.1212	3.8911	3.7283	3.6065	3.5118	3.4358	3.3736
13	6.4143	4.9653	4.3472	3.9959	3.7667	3.6043	3.4827	3.3880	3.3120	3.2497
14	6.2979	4.8567	4.2417	3.8919	3.6634	3.5014	3.3799	3.2853	3.2093	3.1469
15	6.1995	4.7650	4.1528	3.8043	3.5764	3.4147	3.2934	3.1987	3.1227	3.0602
16	6.1151	4.6867	4.0768	3.7294	3.5021	3.3406	3.2194	3.1248	3.0488	2.9862
17	6.0420	4.6189	4.0112	3.6648	3.4379	3.2767	3.1556	3.0610	2.9849	2.9222
18	5.9781	4.5597	3.9539	3.6083	3.3820	3.2209	3.0999	3.0053	2.9291	2.8664
19	5.9216	4.5075	3.9034	3.5587	3.3327	3.1718	3.0509	2.9563	2.8801	2.8172
20	5.8715	4.4613	3.8587	3.5147	3.2891	3.1283	3.0074	2.9128	2.8365	2.7737
21	5.8266	4.4199	3.8188	3.4754	3.2501	3.0895	2.9686	2.8740	2.7977	2.7348
22	5.7863	4.3828	3.7829	3.4401	3.2151	3.0546	2.9338	2.8392	2.7628	2.6998
23	5.7498	4.3492	3.7505	3.4083	3.1835	3.0232	2.9023	2.8077	2.7313	2.6682
24	5.7166	4.3187	3.7211	3.3794	3.1548	2.9946	2.8738	2.7791	2.7027	2.6396
25	5.6864	4.2909	3.6943	3.3530	3.1287	2.9685	2.8478	2.7531	2.6766	2.6135
26	5.6586	4.2655	3.6697	3.3289	3.1048	2.9447	2.8240	2.7293	2.6528	2.5896
27	5.6331	4.2421	3.6472	3.3067	3.0828	2.9228	2.8021	2.7074	2.6309	2.5676
28	5.6096	4.2205	3.6264	3.2863	3.0626	2.9027	2.7820	2.6872	2.6106	2.5473
29	5.5878	4.2006	3.6072	3.2674	3.0438	2.8840	2.7633	2.6686	2.5919	2.5286
30	5.5675	4.1821	3.5894	3.2499	3.0265	2.8667	2.7460	2.6513	2.5746	2.5112
40	5.4239	4.0510	3.4633	3.1261	2.9037	2.7444	2.6238	2.5289	2.4519	2.3882
60	5.2856	3.9253	3.3425	3.0077	2.7863	2.6274	2.5068	2.4117	2.3344	2.2702
120	5.1523	3.8046	3.2269	2.8943	2.6740	2.5154	2.3948	2.2994	2.2217	2.1570
inf	5.0239	3.6889	3.1161	2.7858	2.5665	2.4082	2.2875	2.1918	2.1136	2.0483

F Table for alpha=.025 (Cont.)

df2/ df1	12	15	20	24	30	40	60	120	INF
1	976.7079	984.8668	993.1028	997.2492	1001.414	1005.598	1009.800	1014.020	1018.258
2	39.4146	39.4313	39.4479	39.4562	39.465	39.473	39.481	39.490	39.498
3	14.3366	14.2527	14.1674	14.1241	14.081	14.037	13.992	13.947	13.902
4	8.7512	8.6565	8.5599	8.5109	8.461	8.411	8.360	8.309	8.257
5	6.5245	6.4277	6.3286	6.2780	6.227	6.175	6.123	6.069	6.015
6	5.3662	5.2687	5.1684	5.1172	5.065	5.012	4.959	4.904	4.849
7	4.6658	4.5678	4.4667	4.4150	4.362	4.309	4.254	4.199	4.142
8	4.1997	4.1012	3.9995	3.9472	3.894	3.840	3.784	3.728	3.670
9	3.8682	3.7694	3.6669	3.6142	3.560	3.505	3.449	3.392	3.333
10	3.6209	3.5217	3.4185	3.3654	3.311	3.255	3.198	3.140	3.080
11	3.4296	3.3299	3.2261	3.1725	3.118	3.061	3.004	2.944	2.883
12	3.2773	3.1772	3.0728	3.0187	2.963	2.906	2.848	2.787	2.725
13	3.1532	3.0527	2.9477	2.8932	2.837	2.780	2.720	2.659	2.595
14	3.0502	2.9493	2.8437	2.7888	2.732	2.674	2.614	2.552	2.487
15	2.9633	2.8621	2.7559	2.7006	2.644	2.585	2.524	2.461	2.395
16	2.8890	2.7875	2.6808	2.6252	2.568	2.509	2.447	2.383	2.316
17	2.8249	2.7230	2.6158	2.5598	2.502	2.442	2.380	2.315	2.247
18	2.7689	2.6667	2.5590	2.5027	2.445	2.384	2.321	2.256	2.187
19	2.7196	2.6171	2.5089	2.4523	2.394	2.333	2.270	2.203	2.133
20	2.6758	2.5731	2.4645	2.4076	2.349	2.287	2.223	2.156	2.085
21	2.6368	2.5338	2.4247	2.3675	2.308	2.246	2.182	2.114	2.042
22	2.6017	2.4984	2.3890	2.3315	2.272	2.210	2.145	2.076	2.003
23	2.5699	2.4665	2.3567	2.2989	2.239	2.176	2.111	2.041	1.968
24	2.5411	2.4374	2.3273	2.2693	2.209	2.146	2.080	2.010	1.935
25	2.5149	2.4110	2.3005	2.2422	2.182	2.118	2.052	1.981	1.906
26	2.4908	2.3867	2.2759	2.2174	2.157	2.093	2.026	1.954	1.878
27	2.4688	2.3644	2.2533	2.1946	2.133	2.069	2.002	1.930	1.853
28	2.4484	2.3438	2.2324	2.1735	2.112	2.048	1.980	1.907	1.829
29	2.4295	2.3248	2.2131	2.1540	2.092	2.028	1.959	1.886	1.807
30	2.4120	2.3072	2.1952	2.1359	2.074	2.009	1.940	1.866	1.787
40	2.2882	2.1819	2.0677	2.0069	1.943	1.875	1.803	1.724	1.637
60	2.1692	2.0613	1.9445	1.8817	1.815	1.744	1.667	1.581	1.482
120	2.0548	1.9450	1.8249	1.7597	1.690	1.614	1.530	1.433	1.310
inf	1.9447	1.8326	1.7085	1.6402	1.566	1.484	1.388	1.268	1.000

F Table for alpha=.01

df2/ df1	1	2	3	4	5	6	7	8	9	10
1	4052.181	4999.500	5403.352	5624.583	5763.650	5858.986	5928.356	5981.070	6022.473	6055.847
2	98.503	99.000	99.166	99.249	99.299	99.333	99.356	99.374	99.388	99.399
3	34.116	30.817	29.457	28.710	28.237	27.911	27.672	27.489	27.345	27.229
4	21.198	18.000	16.694	15.977	15.522	15.207	14.976	14.799	14.659	14.546
5	16.258	13.274	12.060	11.392	10.967	10.672	10.456	10.289	10.158	10.051
6	13.745	10.925	9.780	9.148	8.746	8.466	8.260	8.102	7.976	7.874
7	12.246	9.547	8.451	7.847	7.460	7.191	6.993	6.840	6.719	6.620
8	11.259	8.649	7.591	7.006	6.632	6.371	6.178	6.029	5.911	5.814
9	10.561	8.022	6.992	6.422	6.057	5.802	5.613	5.467	5.351	5.257
10	10.044	7.559	6.552	5.994	5.636	5.386	5.200	5.057	4.942	4.849
11	9.646	7.206	6.217	5.668	5.316	5.069	4.886	4.744	4.632	4.539
12	9.330	6.927	5.953	5.412	5.064	4.821	4.640	4.499	4.388	4.296
13	9.074	6.701	5.739	5.205	4.862	4.620	4.441	4.302	4.191	4.100
14	8.862	6.515	5.564	5.035	4.695	4.456	4.278	4.140	4.030	3.939
15	8.683	6.359	5.417	4.893	4.556	4.318	4.142	4.004	3.895	3.805
16	8.531	6.226	5.292	4.773	4.437	4.202	4.026	3.890	3.780	3.691
17	8.400	6.112	5.185	4.669	4.336	4.102	3.927	3.791	3.682	3.593
18	8.285	6.013	5.092	4.579	4.248	4.015	3.841	3.705	3.597	3.508
19	8.185	5.926	5.010	4.500	4.171	3.939	3.765	3.631	3.523	3.434
20	8.096	5.849	4.938	4.431	4.103	3.871	3.699	3.564	3.457	3.368
21	8.017	5.780	4.874	4.369	4.042	3.812	3.640	3.506	3.398	3.310
22	7.945	5.719	4.817	4.313	3.988	3.758	3.587	3.453	3.346	3.258
23	7.881	5.664	4.765	4.264	3.939	3.710	3.539	3.406	3.299	3.211
24	7.823	5.614	4.718	4.218	3.895	3.667	3.496	3.363	3.256	3.168
25	7.770	5.568	4.675	4.177	3.855	3.627	3.457	3.324	3.217	3.129
26	7.721	5.526	4.637	4.140	3.818	3.591	3.421	3.288	3.182	3.094
27	7.677	5.488	4.601	4.106	3.785	3.558	3.388	3.256	3.149	3.062
28	7.636	5.453	4.568	4.074	3.754	3.528	3.358	3.226	3.120	3.032
29	7.598	5.420	4.538	4.045	3.725	3.499	3.330	3.198	3.092	3.005
30	7.562	5.390	4.510	4.018	3.699	3.473	3.304	3.173	3.067	2.979
40	7.314	5.179	4.313	3.828	3.514	3.291	3.124	2.993	2.888	2.801
60	7.077	4.977	4.126	3.649	3.339	3.119	2.953	2.823	2.718	2.632
120	6.851	4.787	3.949	3.480	3.174	2.956	2.792	2.663	2.559	2.472
inf	6.635	4.605	3.782	3.319	3.017	2.802	2.639	2.511	2.407	2.321

F Table for alpha=.01 (Cont.)

df2/ df1	12	15	20	24	30	40	60	120	INF
1	6106.321	6157.285	6208.730	6234.631	6260.649	6286.782	6313.030	6339.391	6365.864
2	99.416	99.433	99.449	99.458	99.466	99.474	99.482	99.491	99.499
3	27.052	26.872	26.690	26.598	26.505	26.411	26.316	26.221	26.125
4	14.374	14.198	14.020	13.929	13.838	13.745	13.652	13.558	13.463
5	9.888	9.722	9.553	9.466	9.379	9.291	9.202	9.112	9.020
6	7.718	7.559	7.396	7.313	7.229	7.143	7.057	6.969	6.880
7	6.469	6.314	6.155	6.074	5.992	5.908	5.824	5.737	5.650
8	5.667	5.515	5.359	5.279	5.198	5.116	5.032	4.946	4.859
9	5.111	4.962	4.808	4.729	4.649	4.567	4.483	4.398	4.311
10	4.706	4.558	4.405	4.327	4.247	4.165	4.082	3.996	3.909
11	4.397	4.251	4.099	4.021	3.941	3.860	3.776	3.690	3.602
12	4.155	4.010	3.858	3.780	3.701	3.619	3.535	3.449	3.361
13	3.960	3.815	3.665	3.587	3.507	3.425	3.341	3.255	3.165
14	3.800	3.656	3.505	3.427	3.348	3.266	3.181	3.094	3.004
15	3.666	3.522	3.372	3.294	3.214	3.132	3.047	2.959	2.868
16	3.553	3.409	3.259	3.181	3.101	3.018	2.933	2.845	2.753
17	3.455	3.312	3.162	3.084	3.003	2.920	2.835	2.746	2.653
18	3.371	3.227	3.077	2.999	2.919	2.835	2.749	2.660	2.566
19	3.297	3.153	3.003	2.925	2.844	2.761	2.674	2.584	2.489
20	3.231	3.088	2.938	2.859	2.778	2.695	2.608	2.517	2.421
21	3.173	3.030	2.880	2.801	2.720	2.636	2.548	2.457	2.360
22	3.121	2.978	2.827	2.749	2.667	2.583	2.495	2.403	2.305
23	3.074	2.931	2.781	2.702	2.620	2.535	2.447	2.354	2.256
24	3.032	2.889	2.738	2.659	2.577	2.492	2.403	2.310	2.211
25	2.993	2.850	2.699	2.620	2.538	2.453	2.364	2.270	2.169
26	2.958	2.815	2.664	2.585	2.503	2.417	2.327	2.233	2.131
27	2.926	2.783	2.632	2.552	2.470	2.384	2.294	2.198	2.097
28	2.896	2.753	2.602	2.522	2.440	2.354	2.263	2.167	2.064
29	2.868	2.726	2.574	2.495	2.412	2.325	2.234	2.138	2.034
30	2.843	2.700	2.549	2.469	2.386	2.299	2.208	2.111	2.006
40	2.665	2.522	2.369	2.288	2.203	2.114	2.019	1.917	1.805
60	2.496	2.352	2.198	2.115	2.028	1.936	1.836	1.726	1.601
120	2.336	2.192	2.035	1.950	1.860	1.763	1.656	1.533	1.381
inf	2.185	2.039	1.878	1.791	1.696	1.592	1.473	1.325	1.000

Helping Formulas – 2005 :	
$\frac{\bar{x} - \mu_o}{s / \sqrt{n}}, \frac{\bar{d} - \bar{D}_o}{s_d / \sqrt{n}}$	$\bar{P} = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$
$\bar{x} \pm t_{(\alpha/2, n-1)} \frac{s}{\sqrt{n}}$	$(p_1 - p_2) \pm Z_{\alpha/2} \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$
$\frac{p - \pi_o}{\sqrt{\frac{\pi_o(1-\pi_o)}{n}}}$	$S_{Y X} = \sqrt{\frac{\sum Y^2 - \hat{a} \sum Y - \hat{b} \sum XY}{n-2}}$
$p \pm z_{\alpha/2} \sqrt{\frac{p(1-p)}{n}}$	$R^2 = \frac{\hat{a} \sum Y + \hat{b} \sum XY - n \bar{Y}^2}{\sum Y^2 - n \bar{Y}^2}$
$\frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_o}{\sqrt{S_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$	$LSD = \sqrt{MSE} \cdot \sqrt{\frac{2}{n}} \cdot t_{(2(n-1), \alpha/2)}$
$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1 + n_2 - 2}$	$\hat{Y} = \hat{a} + \hat{b}X$
$\frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_o}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$	$\hat{b}_{Y X} = \frac{\sum XY - n \bar{X} \bar{Y}}{\sum X^2 - n \bar{X}^2}$
$(\bar{x}_1 - \bar{x}_2) \pm Z_{\alpha/2} \sqrt{S_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}$	$\hat{a} = \bar{Y} - \hat{b} \bar{X}, \bar{X} = \frac{\sum X}{n}, \bar{Y} = \frac{\sum Y}{n}$
$(\bar{x}_1 - \bar{x}_2) \pm t_{(\alpha/2, n_1 + n_2 - 2)} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$	Expected Frequency = $\frac{\text{row sum} \times \text{col sum}}{\text{Total Sum}},$ $\chi^2 = \sum_{\forall \text{cells}} \frac{(O_i - E_i)^2}{E_i} \mid cc = \sqrt{\frac{\chi^2}{\chi^2 + n}}$
$\bar{d} = \frac{\sum d}{n}, S_d = \sqrt{\frac{\sum d_i^2 - n \bar{d}^2}{n-1}}$	Normal Value for Sign Test = $\frac{2R-n}{\sqrt{n}}, R = \#(+) \mid$
$\bar{d} \pm t_{(\alpha/2, n-1)} \frac{S_d}{\sqrt{n}}$	$Z = \frac{V - \frac{n(n+1)}{4}}{\sqrt{n(n+1)(2n+1)/24}},$ $V = \text{Sum of (+) ranks}$
$\frac{(p_1 - p_2) - (\pi_1 - \pi_2)_0}{\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}}$	$r_s = 1 - \frac{6 \sum d_i^2}{n(n^2-1)}$
$\frac{(p_1 - p_2) - (\pi_1 - \pi_2)_0}{\sqrt{\bar{p}(1-\bar{p}) \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$	$r_p = \frac{\sum XY - n \bar{X} \bar{Y}}{\sqrt{(\sum X^2 - n \bar{X}^2)(\sum Y^2 - n \bar{Y}^2)}}$

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